系所： ＿＿數學系＿＿＿＿ 組別： ＿甲組選考甲＿ 科目： ＿高等微積分＿

☆☆請在答案紙上作答☆☆ 共1頁，第1頁

1. (10%) Prove, by the $\varepsilon - \delta$ definition of limits, that $\lim\limits_{x \to -1} \dfrac{1}{\sqrt{3x+7}} = \dfrac{1}{2}$.

2. (40%, 10% each) Evaluate the following integrals and limits.

(1) $\displaystyle\int_0^1 e^{\sqrt{x}} dx$ (2) $\displaystyle\int_0^1 \ln x \, dx$ (3) $\lim\limits_{x \to 0} (\dfrac{\sin x}{x})^{\frac{1}{x^2}}$ (4) $\lim\limits_{h \to 0} \dfrac{1}{h} \displaystyle\int_3^{3+h} e^{-x^2} dx$

3. (20%) Let $a_n \geq 0$ and $\sum a_n$ converge. Do the series $\displaystyle\sum_{n=1}^{\infty} a_n^2$ and $\sum \sqrt{a_n}$ converge or diverge ?

Prove your answer.

4. (10%) Does the series $\displaystyle\sum_{n=1}^{\infty} \dfrac{\sin nx}{n^2}$ converge uniformly on R ? Prove your answer.

5. (20%) Let $f(x, y) = \begin{cases} \dfrac{x^3 + y^3}{x^2 + y^2} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$

Find $\dfrac{\partial f}{\partial x}(0,0), \ \dfrac{\partial f}{\partial y}(0,0)$ and the directional derivative of f at $(0, 0)$ in the direction of unit vector

$u = (u_1, u_2)$. Is $f(x, y)$ continuous at $(0, 0)$? Prove your answer.