國立彰化師範大學103學年度博士班招生考試試題

系所： 數學系　　組別： 甲組（選考甲）　　科目： 高等微積分

☆☆請在答案紙上作答☆☆　　　　共 1 頁，第 1 頁

1. Let f be continuous on $[a,b]$ and let $M=\sup\{\,|f(x)|\,|\,x\in[a,b]\}$.

Show that $M=\lim_{n\to\infty}\{\int_a^b |f(x)|^n\}^{\frac{1}{n}}$. (20%)

2. (a) Let $a_n \ge 0$ and $\sum a_n$ converge. Do the series $\sum_{n=1}^{\infty} a_n^2$ and $\sum \sqrt{a_n}$ converge or diverge ? Prove your answer. (12%)

(b) Does the series $\sum_{n=1}^{\infty} \frac{\tan^{-1} nx}{n^{1.01}}$ converge for each $x\in R$ and converge uniformly on R ? Prove your answers. (8%)

3. Let $f(x,y)=\begin{cases} \dfrac{\ln(1+xy)}{x} & x\neq 0 \\ y & x=0 \end{cases}$ $(x,y)\in D=\{(x,y)\in R^2 \mid xy>-1\}$.

Prove that f is continuous on D. (20%)

4. (a) Suppose that $\{f_n\}$ is a sequence of continuous functions on $D\subseteq R^p$ into R^q, and the sequence converges uniformly on D to a function f. Show that f is continuous on D. (15%)

(b) Show, by an example, that if the convergence in (a) is not uniform, then f may not be continuous. (5%)

5. Suppose that f is a continuous real-valued function on $[0,1]$ satisfying

$\int_0^1 f(x)x^n dx = 0$ for all $n=0,1,2,\cdots$.

Prove that $f(x)\equiv 0$ on $[0,1]$. (20%)