

國立彰化師範大學 102 學年度博士班招生考試試題

系所：數學系

選考甲

科目：高等微積分

☆☆請在答案紙上作答☆☆

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1. Suppose that $f(x) = \sum_{k=1}^{\infty} \frac{\cos(kx)}{k^2}$. Calculate $\int_0^{\pi/2} f(x) dx$ (20%)

2. Evaluate $\lim_{n \rightarrow \infty} \int_2^9 \left(1 + \frac{x}{n}\right)^n e^{-x} dx$. (20%)

3. Let $F(x) = \int_{x^2}^{x^3} (t^2 + 1) dt$. Find $F'(2)$. (20%)

4. If $f(x)$ is continuous on $[a, b]$, prove that f is Riemann integrable on $[a, b]$. (20%)

5. Prove that $\int_1^{\infty} \frac{\sin x}{x} dx$ exists. (20%)