

國立彰化師範大學 101 學年度博士班招生考試試題

系所：數學系

科目：高等微積分

☆☆請在答案卷上作答☆☆

第 1 頁，共 1 頁

每題二十分，共五題

1. If E is closed in R^n and $a \notin E$, prove that $\inf_{x \in E} \|x - a\| > 0$.
2. If $|a_n| \leq 8$ for all $n \in N$, prove that $\sum_{n=0}^{\infty} a_n x^n$ has a positive radius of convergence.
3. Evaluate $\lim_{n \rightarrow \infty} \int_1^3 \frac{nx^{12} + 5}{x^3 + nx^8} dx$. Give a proof for your answer.
4. Suppose that $f_n \rightarrow f$ uniformly on a closed interval $[a, b]$. If each f_n is Riemann integrable on $[a, b]$, prove that f is Riemann integrable on $[a, b]$.
5. Suppose that $E \subseteq R^n$ is connected in R^n . Prove that \bar{E} is connected in R^n .
 \bar{E} is the closure of E .