國立彰化師範大學 96 學年度博士班招生考試試題

系所：數學系　　　　科目：高等微積分

☆☆請在答案紙上作答☆☆　　　　

1. (14%)

(a) If F is a closed set in R and if $d(x,F)=\inf\{\|x-z\|: z\in F\}=0$. Show $x\in F$.

(b) If f is continuous and has period 2π, show that it can be approximated by a trigonometric polynomials.

2. (14%)

(a) If $f(x)=x^3$ for $x\in R$. Prove that f is not uniformly continuous on R.

(b) Let (f_n) be a sequence of real valued functions defined on [0,1] and assumed that

$f_n \to f$ uniformly on [0 ,1]. Prove or disprove $\lim_{n\to\infty}\int_0^{1-\frac{1}{n}} f_n(x)dx=\int_0^1 f(x)dx$.

3. (14%)

(a) Let f be the function on R to R defined by

$$\begin{cases} x & x \quad rational \\ 1-x & x \quad irrational \end{cases}$$

Show that f is continuous at $x=\frac{1}{2}$ and discontinuous everywhere.

(b)If $K\subseteq D(F)=$ domain of function f, and f is continuous on K, show that $f(K)$ is compact.

4. (14%)

(a) $f(x)=\dfrac{1}{1+x^2}$, show that f is uniformly continuous on R.

(b) Show that $\int_0^\infty e^{-x^2}dx=\dfrac{\sqrt{\pi}}{2}$

5. (14%)

Let $f: R^p \to R^p$ is a function and if there exists $0<\alpha<1$ such that $\|f(x)-f(y)\|\le\alpha\|x-y\|$ for all $x,y\in R^p$. Show that f has a unique fixed point.

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6. (10%)

(a) Let f be continuous and such that $f(x)\ge 0$ for all $x\in[a,b]$. If g is strictly increasing on $[a,b]$ and $\int_a^b f(x)g(x)dx=0$. Show that $f(x)=0$ for all $x\in[a,b]$.

(b) If $f:[a,b]\times[c,d]\to R$ is a continuous function and g is Riemann integrable on $[a,b]$ and $F:[a,b]\to R$ is defined by $f(t)=\int_a^b f(x,t)g(x)dx$. Show that F is continuous on $[c,d]$.

7. (10%)

let $f:R^2\to R$ be defined by

$$f(x,y)=\begin{cases}\dfrac{xy(x^2-y^2)}{x^2+y^2} & (x,y)\ne(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$$

Show that $\dfrac{\partial f}{\partial x},\dfrac{\partial f}{\partial y}$ are continuous at (0,0), and show that $\dfrac{\partial^2 f(0,0)}{\partial y\partial x}\ne\dfrac{\partial^2 f(0,0)}{\partial x\partial y}$.

8. (10%)

If f_n is continuous on $D\subseteq R^p$ to R^q for each n and if $\{f_n\}$converges to f uniformly on D. Show that f is continuous on D.