國立彰化師範大學 99 學年度博士班招生考試試題

系所：數學系　　選考乙　　科目：代數

☆☆請在答案紙上作答☆☆　　

1. Prove that the group of all rational numbers under addition can not be written as a direct sum of two nontrivial subgroups. (15%)

2. Let G be a group of order 56. Is G a simple group ? Explain ! (15%)

3. Let R be a ring with identity. Suppose that I and J are ideals of R and $R = I + J$. Show that $R/(I \cap J) \cong R/I \oplus R/J$. (15%)

4. Let R be an integral domain with identity and let $M_n(R)$ denote the $n \times n$ matrix ring over R. Show that if I and J are ideals of $M_n(R)$ such that $ab = 0$ for all $a \in I$ and $b \in J$, then $I = 0$ or $J = 0$. (15%)

5. (a) Let F be a finite field. Prove that $F \setminus \{0\}$ is a cyclic group under multiplication. (7%)

   (b) Construct a finite filed of order 8. (7%)

   (c) Let R be a finite ring with identity. Show that if R has exactly p elements, where p is a prime number, then R is a field. (6%)

6. A field is called perfect if every finite extension is a separable extension. Prove that if F is a field of characteristic $p > 0$, then F is perfect if and only if for any $a \in F$, there exists $b \in F$ such that $b^p = a$. (20%)