

國立彰化師範大學 98 學年度博士班招生考試試題

系所：數學系

科目：實變數函數論

☆☆請在答案紙上作答☆☆

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下面每題 20 分，共 100 分

1. Give the definition of Borel subsets in R^N .
2. Let Z be a subset with measure zero in R . Show that $A = \{x^2 : x \in Z\}$ has also measure zero in R .
3. Assume $f : R^N \rightarrow R$ is measurable. Prove that $f^{-1}(G)$ is measurable for every open set in R .

Let E be a measurable set in R^N . A sequence $\{f_n\}$ in $L^p(E)$, $1 \leq p < \infty$, is said to converge weakly to a function f in $L^p(E)$ if $\int f_n g \rightarrow \int fg$ for all $g \in L^{p'}(E)$, where $\frac{1}{p} + \frac{1}{p'} = 1$.

4. Prove that if $f_n \rightarrow f$ in L^p norm, then $\{f_n\}$ converges weakly to f in $L^p(E)$.
5. If $f, g \in L^3(E)$, prove $f + g \in L^3(E)$.