# 國立彰化師範大學 96 學年度博士班招生考試試題

系所：數學系　　　　　　　　　　　　　　　　　科目：實變數函數論

☆☆請在答案紙上作答☆☆　　　　　　　　　　　　　　　共 1 頁，第 1 頁

**Answer all the questions.**

1. (20%) Let A be the subset of [0,1] which consists of all numbers which do not have the digits 4 appearing in their decimal expansion. Find the Lebesgue measure of A.

2. (20%) Let f be a measurable finite-valued function on [0,1], and suppose that $|f(x)-f(y)|$ is integrable on $[0,1]\times[0,1]$. Show that f(x) is integrable on [0,1]. Is it true if [0,1] is replaced by R?

3. (20%) Consider the function defined over R by

$$f(x)=\begin{cases} x^{-1/2} & , \quad \text{if } 0<x<1 \\ 0 & , \quad \text{otherwise} \end{cases}.$$

For a fixed enumeration $\{r_n\}_{n=1}^{\infty}$ of the rationals Q, let $F(x)=\sum_{n=1}^{\infty}2^{-n}f(x-r_n)$.

Prove that F is measurable and integrable. Hence the series defining F converges for almost every x in R.

4. (20%) Establish the following relations between $L^2(R)$ and $L^1(R)$:

(a) Neither the inclusion $L^2(R)\subseteq L^1(R)$ nor $L^1(R)\subseteq L^2(R)$ is valid.

(b) If $f\in L^2(R)$ is supported on a measurable set E of finite measure, then f is integrable.

(c) If a bounded function f is integrable, then it is square integrable.

5. (20%) Let f be integrable on R. For real δ, define $f_\delta(x)=f(\delta x)$. Show that

$f_\delta \to f$ in the $L^1-$norm as $\delta \to 1^+$.