國立彰化師範大學104學年度博士班招生考試試題

系所： 數學系　　組別： 乙組(統計組)　　科目： 統計學

☆☆請在答案紙上作答☆☆　　

1. Let $X_1, X_2, \ldots, X_n$ be a random sample with the following probability density function

$$f(x \mid \theta) = \theta x^{-(\theta+1)}, \quad x > 1, \theta > 0.$$

Suppose that $\tau(\theta) = \frac{1}{\theta}$ is a function of θ.

(a) Find the maximum likelihood estimator (MLE) of $\tau(\theta)$. (10%)

(b) Show that the answer in (a) is an unbiased estimator of $\tau(\theta)$. (10%)

(c) Is the answer in (a) an uniformly minimum variance unbiased estimator (UMVUE) of $\tau(\theta)$?

Justify your answer. (10%)

2. For a comparative study of two treatments *A* and *B*, treatment *A* is applied to the *n* units and treatment *B* to the other *m* units, where the response measurements for treatment *A* and treatment *B* are respectively recorded as $X_{11},\ldots,X_{1n}$ and $X_{21},\ldots,X_{2m}$. These data constitute independent random samples from two populations. We wish to test the null hypothesis H_0: *A* and *B* populations are identical versus the alternative hypothesis H_1: Population *A* is shifted from population *B* toward larger values. Please give a reasonable statistical method to solve this problem. (20%)

3. Please clearly describe the definition of a receiver operating characteristic (ROC) curve for a binary classifier. In addition, please plot a ROC curve with specifying the X-axis and Y-axis labels for illustration. (20%)

4. Consider the following models:

Model 1: simple logistic regression $\ln\frac{p}{1-p}=\beta_0+\beta_1 x$,

Model 2: simple linear regression $y=\beta_0+\beta_1 x+\varepsilon$,

where p is the expectation of a binary response, y is the continuous response and $\varepsilon \sim^{iid} N(0,\sigma^2)$.

(a) What are the differences between Model 1 and 2? And why Model 1 does not have the error term ε like that in the model 2? (10%)

(b) The following table shows the outputs of the simple logistic regression Model 1 for the sample size n=24:

Analysis of Maximum Likelihood Estimates					
Parameter	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	1	-4.4449	1.8432	5.8156	0.0159
x	1	1.9244	0.9116	4.4568	0.0348

Please calculate and interpret $\exp(\hat{\beta}_1)$, and calculate the 95% confidence limits for $\exp(\beta_1)$. (12%)

(c) Based on the above table, please test the null hypothesis $H_0:\beta_1=0$, and write down the alternative hypothesis, the test used, and your conclusion using a 5% level of significance. (8%)

Note: $Z_{0.95}=1.645$; $Z_{0.975}=1.96$

$\chi^2_{1,0.95}=3.841$; $\chi^2_{2,0.95}=5.991$; $\chi^2_{3,0.95}=7.815$; $\chi^2_{4,0.95}=9.488$

$\chi^2_{1,0.975}=5.024$; $\chi^2_{2,0.975}=7.378$; $\chi^2_{3,0.975}=9.348$; $\chi^2_{4,0.975}=11.143$