# 國立彰化師範大學 **101** 學年度<u>博士班</u>招生考試試題

系所： <u>數學系</u>                                           科目： <u>機率與統計</u>

☆☆請在答案卷上作答☆☆                                      第 1 頁，共 2 頁

1. (20%) State and prove the Central Limit Theorem for i.i.d. random variables $(X_n, n \geq 1)$ with $E(X_1) = \mu$ and $var(X_1) = \sigma^2$.

2. (20%) Determine the likelihood ratio test for testing the hypothesis $H_0 : \theta = 0$ vs. $H_a : \theta \neq 0$ at level of significance $\alpha$ on the basis of one observation from the p.d.f. $f(x; \theta) = \dfrac{1}{\pi[1+(x-\theta)^2]}$, for $x \in \Re$, $\theta > 0$.

3. (20%) Let $X_1, \cdots, X_n$ be normally distributed random variables with mean $\mu$ and variance $\sigma^2 < \infty$. Show that $X_1, \cdots, X_n$ are independent if an only if they are uncorrelated.

4. 以下是用統計軟體執行迴歸分析所得部分結果。

   The regression equation is $\hat{Y} = -68.86 + 1.4546X_1 + 9.366X_2$

| Predictor | **Coefficient** | **Standard Error** | **T** | **P-value** |
|---|---|---|---|---|
| Constant | ——— | 60.02 | ——— | 0.266 |
| $X_1$ | ——— | 0.2118 | ——— | 0.000 |
| $X_2$ | ——— | 4.064 | ——— | 0.033 |

   S=11.01    R-sq =91.7%

   Analysis of Variance

| Source | DF | SS | MS | F | **P-value** |
|---|---|---|---|---|---|
| Regression | ——— | 24015 | ——— | ——— | 0.000 |
| Error | 18 | ——— | ——— | | |
| Total | ——— | ——— | | | |

   (1) (13%) 請完成上述空白處。
   (2) (4%) 試檢定 $\beta_1$ 與 $\beta_2$ 的顯著性，並說明結論。 $\alpha = 0.01$
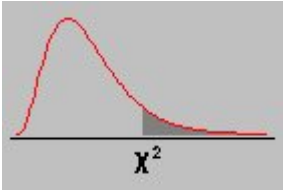   (3) (3%) 試檢定此迴歸模型的顯著性，並說明結論。 $\alpha = 0.01$

5. (20%) To test two methods of instruction, 50 students are selected at random from each of two groups. At the end of the instruction period, each student is assigned a grade (A, B, C, D, or F) by an evaluating team. The data are recorded as follows:

| | Grade | | | | | |
|---|---|---|---|---|---|---|
| | A | B | C | D | F | Totals |
| Group I | 8 | 13 | 16 | 10 | 3 | 50 |
| Group II | 4 | 9 | 14 | 16 | 7 | 50 |

Use a chi-square test with $\alpha = 0.05$ to test whether there is a difference between the two methods of instruction.

**Right tail areas for the *Chi-square* Distribution**



| df\area | .990 | .975 | .950 | .900 | .100 | .050 | .025 | .010 |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.0001 | 0.0009 | 0.0039 | 0.0157 | 2.7055 | 3.8414 | 5.0238 | 6.6349 |
| 2 | 0.0201 | 0.0506 | 0.1025 | 0.2107 | 4.6051 | 5.9914 | 7.3777 | 9.2103 |
| 3 | 0.1148 | 0.2158 | 0.3518 | 0.5843 | 6.2513 | 7.8147 | 9.3484 | 11.3448 |
| 4 | 0.2971 | 0.4844 | 0.7107 | 1.0636 | 7.7794 | 9.4877 | 11.1432 | 13.2767 |
| 5 | 0.5543 | 0.8312 | 1.1454 | 1.6103 | 9.2363 | 11.0705 | 12.8325 | 15.0862 |
| 6 | 0.8720 | 1.2373 | 1.6353 | 2.2041 | 10.6446 | 12.5915 | 14.4493 | 16.8118 |
| 7 | 1.2390 | 1.6898 | 2.1673 | 2.8331 | 12.0170 | 14.0671 | 16.0127 | 18.4753 |
| 8 | 1.6465 | 2.1797 | 2.7326 | 3.4895 | 13.3615 | 15.5073 | 17.5345 | 20.0902 |
| 9 | 2.0879 | 2.7003 | 3.3251 | 4.1681 | 14.6836 | 16.9189 | 19.0227 | 21.6659 |
| 10 | 2.5582 | 3.2469 | 3.9403 | 4.8651 | 15.9871 | 18.3070 | 20.4831 | 23.2092 |