國立彰化師範大學 100 學年度博士班招生考試試題

系所：數學系　　科目：機率與統計

☆☆請在答案紙上作答☆☆　　

1. Let $X_1, X_2, \ldots$ be a sequence of random variables taking values in $\{0,1\}$.

(1) (15%) If $P(X_n = 1) = \frac{1}{n^2}$, prove that $X_n \xrightarrow{a.s.} 0$.

(2) (15%) If $P(X_n = 1) = \frac{1}{n}$ and $X_1, X_2, \ldots$ are independent, prove that $\limsup_{n\to\infty} X_n = 1$ and $\liminf_{n\to\infty} X_n = 0$.

2. (20%) Suppose $X_1, X_2, \ldots$ are independent and each is uniformly distributed in $(0,1)$. Let $R_n = 1_{\left\{X_n \geq \max_{1\leq i\leq n-1} X_i\right\}}$. Prove that $R_1, R_2, \ldots$ are independent and $R_n \sim Bin(1, \frac{1}{n})$.

3. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from a Poisson distribution with mean θ unknown, $\theta > 0$. That is,

$$P_\theta(X = x) = \frac{\theta^x}{x!} e^{-\theta}, \qquad x = 0, 1, 2, \ldots$$

We wish to estimate $g(\theta) = e^{-\theta}$. Consider an estimate $W_n = e^{-\bar{X}_n}$, where $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$.

(1) (10%) Show that W_n is a consistent estimate of $e^{-\theta}$.

(2) (10%) Find the limiting distribution of W_n. That is, find σ^2 so that

$$\sqrt{n}(W_n - e^{-\theta}) \xrightarrow{D} N(0, \sigma^2) \text{ as } n \to \infty.$$

4. A sample of 30 houses that were sold in the last year was taken. The value of the house (Y) was estimated. The independent variables included in the analysis were the number of rooms (X_1), the size of the lot (X_2), the number of bathrooms (X_3), and a dummy variable (X_4), which equals 1 if the house has a garage and equals 0 if the house does not have a garage. The following results were obtained:

	Coefficient	Standard Error
Constant	15,232.5	8,462.5
X_1	2,178.4	778.0
X_2	7.8	2.2
X_3	2,675.2	2,229.3
X_4	1,157.8	463.1

Analysis of Variance

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Squares
Regression		204,242.88	51,060.72
Error		205,890.00	8,235.60

(1) (3%) Write out the estimated equation.

(2) (2%) Interpret the coefficient on the number of rooms (X_1).

(3) (2%) Interpret the coefficient on the dummy variable (X_4).

(4) (3%) Complete the ANOVA table.

(5) (5%) At $\alpha = 0.05$, perform an F test and determine whether or not the regression model is significant.

(6) (5%) Test the significance of β_1 at the 5% level. Be sure to state the null and alternative hypotheses.

(7) (5%) At $\alpha = 0.05$, test to see if there is a significant relationship between sales and the size of the lot.

(8) (3%) Compute the coefficient of determination and interpret its meaning.

(9) (2%) Estimate the value of a house that has 9 rooms, a lot with an area of 7,500, 2 bathrooms, and a garage.

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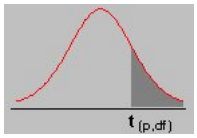
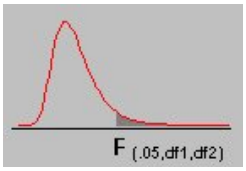
t distribution with right tail probabilities



df\p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834

20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460

F distribution table for $\alpha = 0.05$



df2 / df1	1	2	3	4	5	6	7	8	9	10
1	161.44	199.50	215.70	224.58	230.16	233.98	236.76	238.88	240.54	241.88
2	18.512	19.000	19.164	19.246	19.296	19.329	19.353	19.371	19.384	19.395
3	10.128	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123	8.7855
4	7.7086	6.9443	6.5914	6.3882	6.2561	6.1631	6.0942	6.0410	5.9988	5.9644
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725	4.7351
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.0990	4.0600
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.8660	3.7870	3.7257	3.6767	3.6365
8	5.3177	4.4590	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881	3.3472

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9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789	3.1373
10	4.9646	4.1028	3.7083	3.4780	3.3258	3.2172	3.1355	3.0717	3.0204	2.9782
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962	2.8536
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964	2.7534
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144	2.6710
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458	2.6022
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876	2.5437
16	4.4940	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377	2.4935
17	4.4513	3.5915	3.1968	2.9647	2.8100	2.6987	2.6143	2.5480	2.4943	2.4499
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563	2.4117
19	4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227	2.3779
20	4.3512	3.4928	3.0984	2.8661	2.7109	2.5990	2.5140	2.4471	2.3928	2.3479