

國立彰化師範大學 98 學年度博士班招生考試試題

系所：科學教育研究所 組別：甲 科目：基礎數學(含高微、代數、線性代數)

☆☆請在答案紙上作答☆☆

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1. Prove a rule familiar from junior high school, namely $(-x)(-y) = xy$ for all $x, y \in \mathbf{R}$. (15 分)
2. Show that $\frac{x^n}{n!} \rightarrow 0$ for all $x \in \mathbf{R}$. (15 分)
3. Prove that $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$. (15 分)
4. Let \mathbf{A} be a 2×2 matrix having distinct eigenvalues λ_1 and λ_2 and corresponding eigenvectors \mathbf{v}_1 and \mathbf{v}_2 . Let \mathbf{P} be the matrix whose columns are \mathbf{v}_1 and \mathbf{v}_2 , respectively. Then \mathbf{P} is non-singular and $\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$. (20 分)
5. Let $L: \mathbf{R}^2 \rightarrow \mathbf{R}^1$ defined by $L(x) = x_1 + x_2$. Show L is a linear transformation. (15 分)
6. Let G be a cyclic group and H be a subgroup of G . Show that H is cyclic. (20 分)