

國立彰化師範大學 97 學年度博士班招生考試試題

系所：科學教育研究所

組別：甲組

科目：基礎數學

☆☆請在答案紙上作答☆☆

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1. Let $A = \begin{pmatrix} 5 & -1 & 0 \\ -1 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

(1) Find an invertible matrix P such that $P^{-1}AP$ is diagonal. (10%)

(2) Compute A^n for some positive integer n . (10%)

2. Evaluate $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2-xy+y^2)} dx dy$. (20%)

3. Find the flaws of the following argument which proves the conjecture.

And, correct the conjecture and argument. (20%)

Conjecture: $(D - A) \times (D - B) = (D \times D) - (A \times B)$

Argument:

Let $(x, y) \in (D - A) \times (D - B)$. Then $x \in D - A$ and $y \in D - B$.

Hence, $x \in D$, $x \notin A$, $y \in D$, and $y \notin B$.

Therefore, $(x, y) \in D \times D$ and $(x, y) \notin A \times B$,

and it follows that $(x, y) \in (D - A) \times (D - B)$.

Now let $(x, y) \in (D \times D) - (A \times B)$.

Then $(x, y) \in D \times D$ and $(x, y) \notin A \times B$.

Hence, $x \in D$, $y \in D$, $x \notin A$, and $y \notin B$.

So we have $x \in D - A$ and $y \in D - B$ and, therefore,

$(x, y) \in (D - A) \times (D - B)$.

4. Consider a three-party system with Republicans, Democrats, and Independents. Assume that in the next election, 75% of those who voted Republican again vote Republican, 5% vote Democrat, and 20% vote Independent. Of those who voted Democrat, 20% vote Republican, 60% again vote Democrat, and 20% vote Independent. Of those who voted Independent, 40% vote Republican, 20% vote Democrat, and 40% again vote Independent. Assume these tendencies continue from election to election and that no additional voters enter or leave the system. What is the long-term voting tendency? (Continued ratio could be used to represent this tendency.) (20%)

5. Assume that $f_n \rightarrow f$ uniformly on a subset E of \mathbf{R} and that each f_n is bounded on E . Prove that $\{f_n\}$ is uniformly bounded on E . (20%)