國立彰化師範大學 96 學年度博士班招生考試試題

系所：科學教育研究所　　組別：甲組　　科目：基礎數學

☆☆請在答案紙上作答☆☆　　

1. (20%) Let A be a 2×2 matrix. Prove that, if v_1 and v_2 are eigenvectors of A corresponding to distinct eigenvalues, λ1 and λ2, respectively, then the set $\{v_1, v_2\}$ is linearly independent.

2. (10%) Find an orthogonal basis for the plane $2x - y + z = 0$ in R^3.

3. (15%) The definition of a fine function is that it has a root (zero) at each integer.
   (a) Give an example of a fine function and explain why it is a fine function.
   (b) Give an example of a function which is not fine and explain why it is not fine.
   (c) Determine which of the following conjectures are true.
   - ☐ The product of a fine function and any other function is a fine function.
   - ☐ No polynomial is a fine function.
   - ☐ All trigonometric functions are fine.
   - ☐ All fine functions are periodic.

4. (25%) Read the following arguments and then answer the questions as follows:

> Let the sequence (a_n) be bounded below and decreasing.
> For each $\epsilon > 0$, there is an $n_\epsilon \in N$ such that
> $$a_{n_\epsilon} < \inf_n a_n + \epsilon. \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$
> Thus if $n \geqq n_\epsilon$
> $$a_n < \inf_n a_n + \epsilon. \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$
> If $m \geqq n \geqq n_\epsilon$, we then have
> $$a_n < \inf_n a_n + \epsilon \leqq a_m + \epsilon, \quad \ldots\ldots\ldots\ldots (3)$$
> and so
> $$|a_n - a_m| < \epsilon. \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

   (a) Determine where the fact that (a_n) is bounded below is used in the above.
   ☐(1) ☐(2) ☐(3) ☐(4) ☐none.
   (b) Determine where the fact that (a_n) is decreasing is used in the above.
   ☐(1) ☐(2) ☐(3) ☐(4) ☐none.
   (c) Explain why we can find an $n_\epsilon \in$ N such that (1) holds.
   (d) Express the definition of $\inf a_n$.
   (e) Write down the proposition which can be proved by the above arguments.

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5. (30%) Assume we are considering the survival of whales and that if the number of whales falls below a minimum survival level m, the species will become extinct. Assume also that the population is limited by the carrying capacity M of the environment. Consider the following model for the whale population,

$$\frac{dP}{dt} = k(M-P)(P-m)$$

where P(t) denotes the whale population at time t and k is a positive constant.

(a) List one major assumption implicit in the preceding model.

(b) Graph dP/dt versus P.

(c) Graph P versus t, assuming that the initial population $P(0)=P_0$ satisfies $P_0<m$.

(d) Graph P versus t, assuming that the initial population $P(0)=P_0$ satisfies $m<P_0<M$.

(e) Solve the model given earlier for P as a function of t, assuming that m < P < M for all time.

(f) From part (e), find the limit of P as t approaches infinity.