● 共 100 分，每題配分置於題目後面

1. Find the Laplace transform of the following function. (10%)

$$f(t) = t^2 u(t-1).$$

2. Find the inverse Laplace transform of the following function. (10%)

$$F(s) = \ln \frac{s^2 - 1}{s^2}.$$

3. Using the Laplace transform to solve the initial value problem. (10%)

$$y'' + ty' - y = 0 \quad y(0) = 0, y'(0) = 1.$$

4. Given $A^3 = \begin{bmatrix} 83 & 84 \\ 42 & 41 \end{bmatrix}$,

　(a) Find matrix $A = ?$ (10%)

　(b) Find $A^{20} = ?$ (10%)

5. Find the general solution of the following differential equation. (15%)

$$y'' + 4y' + 4y = e^{-x} \cos x$$

6. Find the particular solution of the following differential equation. (15%)

$$y' + xy = xy^{-1} \quad y(0) = 3$$

7. Find a normal vector of the curve at the given point P. (10%)

$$16x^2 - y^2 = 399, \quad P:(5,1)$$

8. Calculation of the divergence and its value at P. (10%)

$$\vec{v} = \left[0, \sin x^2 yz, \cos xy^2 z \right], \quad P:\left(1, \tfrac{1}{2}, -\pi \right)$$