

國立彰化師範大學九十六學年度碩士班招生考試試題

系所：工業教育與技術學系

組別：乙組

科目：工程數學

請在答案紙上作答

共 2 頁 第 1 頁

1. Find the solution of the following non-homogeneous linear systems (10%)

$$\begin{aligned} y_1' &= 4y_2 \\ y_2' &= 4y_1 + 2 - 16t^2, \quad y_1(0) = 3, \quad y_2(0) = 1 \end{aligned}$$

2. Find a general solution of the non-homogeneous equation (10%)

(a)  $y'' + 2y' + y = e^{-x} \cos x$

(b)  $x^2 y'' - 4xy' + 6y = 21x^{-4}$

3. Solve the given initial value problem (10%)

$$y'' + 3y' + 2y = 1 - u(t-1), \quad y(0) = 0, \quad y'(0) = 1$$

4. Given  $\mathbf{A} = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 \\ 3 & 1 & 3 & 0 & 0 \\ 1 & 2 & 1 & 4 & 0 \\ 2 & 1 & 1 & 1 & 1 \end{vmatrix}$ ,  $\mathbf{B} = \begin{vmatrix} 1 & 2 & 1 & 2 & 1 \\ 0 & 2 & 3 & 3 & 2 \\ 0 & 0 & 3 & 1 & 2 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{vmatrix}$ , Find:  $\det(\mathbf{AB})$ , (5%)

5. Reduce to principal axes. Express  $[x_1, x_2]^T$  in terms of the new variable. (Show the details), (10%)

$$9x_1^2 - 6x_1x_2 + 17x_2^2 = 72$$

6. Given  $\mathbf{A} = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}$ , Find  $\cos(\mathbf{A})$ , (10%)

7. Find the directional derivative of  $f$  at  $P$  in the direction of  $\mathbf{a}$ ,

where  $f = e^x \cos y$ ,  $P: (2, \pi, 0)$ ,  $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$ , (10%)

8. (General Solution) Solve (15%)

$$y'' + w^2 y = r(t)$$

where  $r(t) = \frac{t^2}{4}$ ,  $-\pi < t < \pi$ ,  $r(t + 2\pi) = r(t)$ ,  $|w| \neq 0, 1, 2, \dots$

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9. Solve: (10%)

$$\left\{ \begin{array}{l} \frac{\partial^2 u(x, t)}{\partial t^2} = a^2 \frac{\partial^2 u(x, t)}{\partial x^2} \\ u_x(0, t) = u_x(\ell, t) = u_t(x, 0) = 0 \\ u(x, 0) = e^{-x} \end{array} \right., \text{ where } a \text{ is constant.}$$

10. Solve: (10%)

(a)  $\int_{-\infty}^{\infty} \frac{1}{x^4 + 1} dx = ?$

(b)  $\int_{-\infty}^{\infty} \frac{3x + 2}{x(x - 4)(x^2 + 9)} dx = ?$