1. Consider the fourth order differential equation $y'''' + a_1 y''' + a_2 y'' + a_3 y' + a_4 y = f(t)$, where $a_i = \text{constant}, i = 1, 2, 3, 4$, $y = y(t)$ and $(\ )' = \dfrac{d}{dt}$. By introducing state vector, the fourth order differential equation can be rewritten as a first order state equation (vector equation) $\mathbf{x}' = \mathbf{Ax} + \mathbf{b}$. Please derive the state equation as mentioned. (20%)

2. Rewrite the following equation as a vector equation and find the complete solution of the system:
$$x' + 4x - y = 1$$
$$y' + x + 2y = e^{2t},$$
where $x = x(t)$, $y = y(t)$ and $(\ )' = \dfrac{d}{dt}$. (35%)

3. (a) Find the Laplace transform of the equation $f(t) = f_0(u(t-a) - u(t-b))$, where $u(t)$ is the unit step function; f_0, a, b are positive constants and $a < b$. (10%)

(b) If $f(t)$ has the Laplace transform $F(s)$, then find the inverse transform of $F(s) = \dfrac{e^{-2s}}{(s+3)^2}$. (10%)

4. It is assumed that an arbitrary function $f(t)$ of period $2L$ can be represented by a Fourier series
$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos\frac{n\pi t}{L} + b_n \sin\frac{n\pi t}{L}),$$ where a_0, a_n, and b_n are the coefficients of the series.

(a) Find a_0, a_n, and b_n. (10%)

(b) Prove that the function $f(t)$ can be expressed by the complex exponential form
$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{in\pi t}{L}}, \quad i = \sqrt{-1}.$$ (15%)