

國立彰化師範大學 102 學年度碩士班招生考試試題

系所：機電工程學系、電子工程學系、資訊工程學系積體電路設計碩士班、車輛科技研究所

科目：工程數學

☆☆請在答案紙上作答☆☆

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1. Solve $\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = 3e^t$ as $y(0) = 3$ and $\dot{y}(0) = 0$. (15%)

2. Find $y(t)$ if its Laplace Transform is equal to $\frac{-2s^2 - 2s + 10}{s(s^2 + 3s + 2)}$. (10%)

3. Assume that $f(x)$ has a Fourier sine series $f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$, $0 \leq x \leq L$

a. Show formally that $\frac{2}{L} \int_0^L [f(x)]^2 dx = \sum_{n=1}^{\infty} b_n^2$ (10%)

b. Apply the result of (a) to the Fourier series for the function $f(x) = x$, $-L < x < L$, and thereby

show that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \sum_{n=1}^{\infty} \frac{1}{n^2}$. (15%)

4. Consider the linear system

$$\mathbf{a}\mathbf{x} + \mathbf{y} + \mathbf{z} = 1$$

$$\mathbf{x} + \mathbf{a}\mathbf{y} + \mathbf{z} = 0$$

$$\mathbf{x} + \mathbf{y} + \mathbf{a}\mathbf{z} = 0$$

For what values of \mathbf{a} does the linear system have a unique solution? (25%)

5. Solve the following partial differential equation (25%)

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} \text{ for } 0 < x < L, t > 0$$

$$u(0, t) = u(L, t) = 0 \text{ for } t \geq 0$$

$$u(x, 0) = x(L - x) \text{ for } 0 \leq x \leq L$$