

國立彰化師範大學 101 學年度碩士班招生考試試題

系所： 機電工程學系

組別： 甲、乙組

科目： 工程數學

☆☆請在答案卷上作答☆☆

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1. Solve the following by Laplace Transforms. Show all details:

(1) Solve the IVP $y'' - 6y' + 5y = 29 \cos 2t$, $y(0) = 3.2$, $y'(0) = 6.2$ (10%)

(2) Integral equation $y(t) + \int_0^t y(\tau) \cosh(t - \tau) d\tau = t + e^t$ (10%)

(3) System of ODEs $4y_1' + y_2' - 2y_3' = 0$, (15%)

$$-2y_1' + y_3' = 1,$$

$$2y_2' - 4y_3' = -16t$$

$$y_1(0) = 2, y_2(0) = 0, y_3(0) = 0$$

2. Find the motion of the mass-spring system in fig. 1 with mass **0.25** kg, damping **0**, spring constant **2.25** kg/sec², and driving force $\cos t - 2 \sin t$ nt, assuming zero initial displacement and velocity. For what frequency of the driving force would you get resonance (ω_0)? (25%)

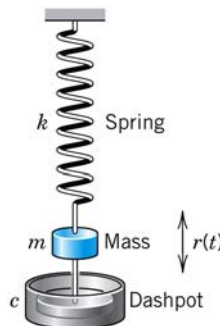


Fig. 1

3. Solve the following heat equation with a source input. Show all details: (20%)

$$\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2} + xt \quad \text{for } 0 < x < \pi, t > 0$$

$$u(0, t) = u(\pi, t) = 0 \quad \text{for } t \geq 0, \text{ and}$$

$$u(x, 0) = f(x) = \begin{cases} 20 & 0 \leq x \leq \pi/4 \\ 0 & \pi/4 < x \leq \pi \end{cases}$$

4. Find the Taylor series of the following function with center $z_0 = 1$. Show the first four terms at least. For what is the convergence of the whole series? (20%)

$$f(z) = \frac{2z^2 + 9z + 5}{z^3 + z^2 - 8z - 12}$$