

國立彰化師範大學 99 學年度碩士班招生考試試題

系所：機電工程學系

科目：工程數學

☆☆請在答案紙上作答☆☆

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1. (30%)

(a) Solve the following differential equation (15%)

$$y'' + 2y' + y = 2x \sin x$$

(b) Solve the following initial differential equation (15%)

$$y'' + 5y' + 6y = \delta(t - \frac{1}{2}\pi) + u(t - \pi) \cos t,$$

$$y(0) = 0, y'(0) = 0$$

where δ is dirac delta function or unit impulse function, and u is unit step function

2. (20%)

$$\text{Define } f(x) = \begin{cases} x, & (0 < x < \pi/2) \\ \pi/2, & (\pi/2 < x < \pi) \end{cases}$$

(a) Calculate the Fourier Cosine series for first five terms (10%)

(b) Calculate the Fourier Sine series for first five terms (10%)

3. (20%)

(a) Explain Divergence theorem and Stoke's theorem (10%)

(b) if $V = \frac{z \sin x \cos y}{e^z \sqrt{x^2 + y^2}}$ compute $(\nabla \times \nabla V)$ (10%)

4. (30%)

(a) What conditions on b_1, b_2, b_3, b_4 , make following system solvable. Find \mathbf{x} in that case

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 2 & 5 & 7 \\ 3 & 9 & 12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \quad (15\%)$$

(b) Diagonalize A , where $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ (5%) and compute A^k to prove the formula

$$A^k = 1/2 \begin{bmatrix} 3^k + 1 & 3^k - 1 \\ 3^k - 1 & 3^k + 1 \end{bmatrix} \quad (10\%)$$