

國立彰化師範大學 98 學年度碩士班招生考試試題

系所：機電工程學系

組別：甲、乙

科目：工程數學

☆☆請在答案紙上作答☆☆

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1. (25%)

(a) Solve the following differential equation (15%)

$$y'' + 4y' + 4y = 3xe^{-2x} + \sin(2x + 2)$$

(b) Solve the following differential equation(10%)

$$x^{-3}y' - x^{-2}y - y^2 = 0$$

2. (25%)

Solve the initial value problem for the system

$$\begin{cases} y_1'' + 8y_1 - 2y_2 = 1 - u(t - 2) \\ y_2'' - 2y_1 + 5y_2 = 0 \end{cases}$$
$$y_1(0) = y_1'(0) = y_2(0) = y_2'(0) = 0$$

Where $u(t-2)$ is an unit step function or Heaviside function

3. (25%)

Evaluate $\iint_S F \cdot n dA$

by (a) direct method (surface integral)(15%)

(b) the divergence theorem (10%)

where

$$F = 4xzi - y^2j + yzk,$$

S is the surface of a cube (正方形) bounded by $x=0, x=1, y=0, y=1, z=0, z=1,$

n is the unit outer normal vector.

4. (25%)

(a) Find the Fourier series of the function $f(x)$ where $f(x) = x^2, -\pi < x < \pi$ (10%)

(b) use the results in (a) to prove $\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$ (10%)

(c) use the results in (a) to calculate $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = ?$ (5%)