國立彰化師範大學 97 學年度碩士班招生考試試題

系所：機電工程學系碩士班　　組別：甲/乙組　　科目：工程數學

☆☆請在答案紙上作答☆☆　　

1. Solve the following three sub-questions:

(a) By using the technique of integration, find the volume of a right circular cone with base radius R and height H. (10%)

(b) By employing the extreme-value theorem, find the volume of a right circular cylinder of greatest volume that can be inserted in the right circular cone as you derived in (a). (10%)

(c) A display screen is to be laid out in a rectangular area and protected by a magnetic traction free wire frame. What is the largest possible area of the display screen if only 500*cm* of the wire is available for the frame? (10%)

Note: Please draw the corresponding diagrams to show your assumptions for the above sub-questions.

2. The trigonometric function $f(t)$ is given by

$$f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos\frac{n\pi t}{p}+b_n\sin\frac{n\pi t}{p}),$$

where $a_0, a_n,$ and b_n are the Euler coefficients of $f(t)$; $2p$ is the period of the function:

(a) Show that the function $f(t)$ can be written as $f(t)=A_0+\sum_{n=1}^{\infty}A_n\sin(\frac{n\pi t}{p}+\theta_n)$. Please express the new defined coefficients, A_0, A_n, θ_n in terms of a_0, a_n, b_n. (15%)

(b) Show that the function $f(t)$ can be written as $f(t)=\sum_{n=-\infty}^{\infty}C_n e^{i\frac{n\pi t}{p}}$, $i=\sqrt{-1}$, e = the natural exponential base. Please express the new defined coefficients, C_n, in terms of a_0, a_n, b_n. (15%)

3. The following three sub-problems are related to the Laplace transformation. It is recalled that the Laplace transform of a function $f(t)$ associates a function of s is given by

$$L[f(t)]=F(s)=\int_0^{\infty}f(t)e^{-st}dt$$

(a) Find the Laplace transform of $f(t)=u(t-a)$, where $u(t-a)=\begin{cases}1, & t\ge a\\ 0, & t<a\end{cases}$. (10%)

(b) Find the inverse transform of $F(s)=\dfrac{e^{-s}}{s^2+3s+2}$. (10%)

(c) By employing Laplace transformation, find the solution of the following differential equation which satisfies the given initial conditions,

$y''+3y'+2y=u(t-1)$, $y=y(t)$ and $y'=\dfrac{dy}{dt}$

with the given conditions $y(0)=0$ and $y'(0)=1$. (20%)