國立彰化師範大學 98 學年度碩士班招生考試試題

系所：電機工程學系 　　　　　　　　　　　　　　　科目：工程數學

1. (30%) Solve the following differential equations

    (a) $\dfrac{1}{x} y' = \ln x + e^x, \quad y(1) = 0$ 　(10%)

    (b) $y'' - 2y' + 5y = xe^{2x}$ 　　(10%)

    (c) $y' - \dfrac{2x}{1-x^2} y = 1, \quad y(0) = 1$ 　(10%)

2. (10%) Solve the equation by Laplace Transform

    $$y'(t) + 5\int_0^t \cos 2u \cdot y(t-u)\,du = 10, \quad y(0) = 2$$

3. (10%) Given $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$, if $f(x)$ at $-\pi < x < \pi$ can be expressed as

    $f(x) = a_0 + a_1 \cos 2x + a_2 \cos 6x + a_3 \cos 10x + \cdots$, find a_0, a_1, a_2, a_3.

4. (10%) Evaluate the following integrals by means of the residue theorem

    (a) $\oint_C \dfrac{1-2z}{z(z-1)(z-3)} dz, \quad C: |z| = 2$ 　(5 %)

    (b) $\oint_C z e^{3/z} dz, \quad C: |z| = 4$ 　(5 %)

5. (10%) Let $A = \begin{bmatrix} 3 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 0 \end{bmatrix}$, find the eigenvalues of Matrix A and their corresponding

    eigenvectors.

6. (15 %) Given a vector function $\vec{F} = \vec{a}_x(3y - c_1 z) + \vec{a}_y(c_2 x - 2z) - \vec{a}_z(c_3 y + z)$, determine

   (a) the constants c_1, c_2, and c_3 if $\nabla \times \vec{F} = 0$. (5 %)

   (b) the scalar potential function V whose negative gradient equals $\vec{F}$, i.e. $-\nabla V = \vec{F}$. (10 %)

7. (15%) If $\vec{F} = xy^2 \vec{a}_x + (x^2 y + y)\vec{a}_y$

   (a) evaluate $\oint \vec{F} \cdot d\vec{\ell}$ along the circumference of a circle of radius 3. (10 %)

   (b) show the Stoke's Theorem. (5 %)