

國立彰化師範大學105學年度碩士班招生考試試題

系所：物理學系(甲組選考甲)、
光電科技研究所(選考甲)

科目：工程數學

☆☆請在答案紙上作答☆☆

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1. Find a general solution of the given equation $y''+2y'+5y = 5x^2 + 4x + 2$. (15%)

2. (a) Find the Fourier series of the function $f(x)$ that is assumed to have period 2π . (10%)

$$f(x) = \begin{cases} 0 & \text{if } 0 < x < \pi \\ 1 & \text{if } \pi < x < 2\pi \end{cases}$$

(b) Find the **complex** Fourier series of the function $f(x)$ that is assumed to have period 2π . (10%)

$$f(x) = \begin{cases} -1 & \text{if } -\pi < x < 0 \\ 1 & \text{if } 0 < x < \pi \end{cases}$$

3. Find the Fourier transform of the function $f(x)$. (15%)

$$f(x) = \begin{cases} x & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$$

4. (a) Find the eigenvalues and corresponding eigenvectors of the matrix \mathbf{A} expressed as (10%)

$$\mathbf{A} = \begin{pmatrix} -3 & 0 & 0 \\ -5 & 6 & -4 \\ -5 & 2 & 0 \end{pmatrix}.$$

(b) Find the general solution of the system (10%)

$$\frac{dx}{dt} = -3x, \quad \frac{dy}{dt} = -5x + 6y - 4z, \quad \frac{dz}{dt} = -5x + 2y$$

5. Solve the initial value problem: $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 13y = 4\delta(t-3)$, $y(0) = y'(0) = 0$ by using the method of the Laplace transform. (15%)

6. Solve the partial differential equation $\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{2} \frac{\partial u(x,t)}{\partial t}$, $0 \leq x < \infty$, $0 \leq t$, satisfying the conditions: $u(x,0) = \frac{1}{10} \sin x$ and $u(0,t) = 0$. (15%)