

# 國立彰化師範大學103學年度碩士班招生考試試題

系所： 光電科技研究所

科目： 工程數學

☆☆請在答案紙上作答☆☆

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1. A resistor  $R$ , an inductor  $L$ , and a capacitor  $C$  are connected in series to a source of voltage  $E(t) = E_0 \sin \omega t$ , where  $t$  is time. The model of this RLC-circuit can be expressed as

$$LI'' + RI' + \frac{1}{C}I = E_0 \omega \cos \omega t$$

Please find the current  $I(t)$  in this RLC-circuit when it is connected to a source of voltage  $E(t) = 155 \sin 377t$  with  $R=100$  ohms,  $L=0.1$  henry, and  $C=10^{-3}$  farad. (20%)

2. Find the Laplace transforms of  $2t^3 e^{-t/2}$ . (10%)

3. (a) Find the Fourier series of the function  $f(x)$ , and period  $p = 2L$ . (10%)

$$f(x) = 0 \quad (-2 < x < 0), \quad f(x) = 2 \quad (0 < x < 2), \quad p = 2L = 4$$

- (b) Find the Fourier transform of the function  $f(x)$ . (10%)

$$f(x) = \begin{cases} e^{2ix} & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

4. (a) Evaluate the value of the definite integral  $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)^2} dx$  by using residue theory. (10%)

- (b) Find the Laurent series of  $f(z) = \sin\left(\frac{z}{1-z}\right)$  expanded about  $z=1$  for  $|z-1| > 0$ . (10%)

5. (a) Obtain eigenvalues and normalized eigenvectors for the following matrix (10%)

$$\mathbf{A} = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- (b) Calculate the matrix function  $f(\mathbf{A}) = 2^{\mathbf{A}}$ . (10%)

6. Solve the first-order partial differential equation,  $x \frac{\partial u}{\partial t} - 2xt \frac{\partial u}{\partial x} = 2tu$ ,  $u(x,0) = x^3$ . (10%)