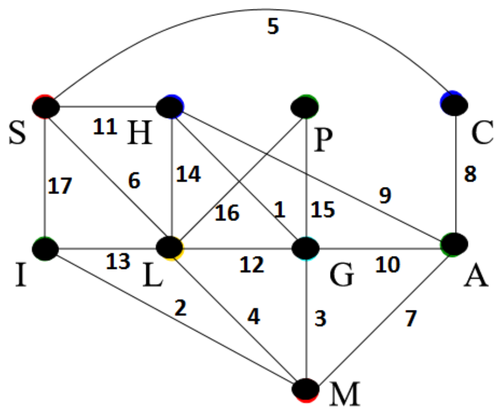
國立彰化師範大學105學年度碩士班招生考試試題

系所：電子工程學系(乙組選考戊)
資訊工程學系(選考乙)
資訊工程學系積體電路設計碩士班(選考戊)

科目：離散數學

☆☆請在答案紙上作答☆☆

1. What is the *chromatic number* of following graph? Given evidence of your answer.(5%)



2. Find the (a) *minimal spanning tree* and (b) *shortest path from node A to node S* of the previous graph. (10%)

3. Show that ***2821*** is a *Carmichael number*. (10%)

4. What sequence of pseudorandom numbers is generated by using the pure multiplication generator $x_{n+1} = (1+4x_n) \bmod 7$ with seed $x_0=3$. (Show the first **5** numbers) (5%)

5. Compute $3^{2003} \bmod 99$. (5%)

6. Find $\gcd(2^{345}-1, 2^{543}-1)$. (5%)

7. Let R be an *equivalence relation* on a set **A**. If x and y are two elements in **A**, then either $[x]=[y]$ or else $[x]\cap[y]=\varnothing$, where $[x]=\{a\in \mathbf{A} \mid (x, a) \in R\}$ is the *equivalence class* of x. (10%)

8. Find the solution to each of these recurrence relations and initial conditions. (10%)
   (a) $a_n = (1-\frac{1}{n+1})a_{n-1}$, $a_0 = 1$.
   (b) $a_n = 5a_{n-1} - 6a_{n-2}$, $a_0 = 1$, $a_1 = 0$.

9. Show that each of the following argument is valid or not? (10%)

   (a)
   $$\begin{array}{l} p \to r \\ q \to r \\ \neg(p \vee q) \\ \hline \therefore \neg r \end{array}$$

   (b)
   $$\begin{array}{l} p \to r \\ q \to r \\ q \vee \neg r \\ \hline \therefore \neg p \end{array}$$

10. If x_1, x_2, x_3, x_4 are nonnegative integers. How many solutions are there? (10%)
   (a) $x_1 + x_2 + x_3 + x_4 = 12$.
   (b) $x_1 + x_2 + x_3 + x_4 < 12$.

11. Let n be an integer. Show that if n^3+5 is odd, then n is even using
   (a) a proof by contraposition. (5%)
   (b) a proof by contradiction. [Hint : try to show $\neg(p \to q)$ is not satisfied. $\neg(p \to q) = p \wedge \neg q$] (5%)

12. Determine the following sets. (2% for each)
   (a) $\varnothing \cup \{\varnothing\}$
   (b) $\{\varnothing\} \cup \{a, \varnothing, \{\varnothing\}\}$
   (c) $\{\varnothing\} \cap \{a, \varnothing, \{\varnothing\}\}$
   (d) $\varnothing \oplus \{a, \varnothing, \{\varnothing\}\}$
   (e) $\{\varnothing\} \oplus \{a, \varnothing, \{\varnothing\}\}$