國立彰化師範大學103學年度碩士班招生考試試題

系所：資訊工程學系　　選考乙　　科目：離散數學

☆☆請在答案紙上作答☆☆　　

每題 10%，共 100%

1. Find the solutions for the following recurrence relations with the initial conditions given. (10%)
(a) $a_n = a_{n-1} + 2a_{n-2}$, $a_0 = 2$ and $a_1 = 7$. (5%)
(b) $a_n = 2na_{n-1}$, $a_0 = 1$. (5%)

2. (a) Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent by developing a series of logical equivalences. (5%)
(b) Construct a truth table for the following compound propositions: $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$. (5%)

3. Determine the following sets. (10%)
(a) $\varnothing \cup \{\varnothing\}$ (2%)
(b) $\{\varnothing\} \cup \{a, \varnothing, \{\varnothing\}\}$ (2%)
(c) $\{\varnothing\} \cap \{a, \varnothing, \{\varnothing\}\}$ (2%)
(d) $\varnothing \oplus \{a, \varnothing, \{\varnothing\}\}$ (2%)
(e) $\{\varnothing\} \oplus \{a, \varnothing, \{\varnothing\}\}$ (2%)

4. Prove that 21 divides $4^{n+1} + 5^{2n-1}$ whenever n is a positive integer. (10%)

5. How many 4-permutations of the positive integers not exceeding 100 contain three consecutive integers k, $k+1$, $k+2$, in the correct order
(a) where these consecutive integers can perhaps be separated by other integers in the permutation? (5%)
(b) where they are in consecutive positions in the permutation? (5%)

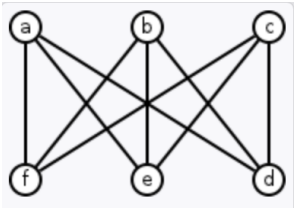
6. Compute 572^{29} **mod** 713. (10%)

7. Let R be the relation on set X={1, 2, 3,4, 5}, where R={(1,1), (1,3), (1,5), (2,2), (2,4), (3,1), (3,3), (3,5), (4,2), (4,4), (5,1), (5,3), (5,5)}. Prove that R is an *equivalence relation*. (10%)

8. For a given graph, if one can walk from node *i* to node *j* along the edges of the graph then we say that there is a *path (or walk)* from *i* to *j*. If we walked on *k* edges, then the walk has length *k*. Given the matrix ***D*** (as follow), which represents the *adjacency matrix* of a graph, determine the number of walks of length *5* from $\boldsymbol{v_2}$ to $\boldsymbol{v_4}$. (10%)

$$D = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

9. A graph is *planar* if it can be drawn in the plane without its edges crossing. Show that the graph $K_{3,3}$ (as follow) is not planar. (10%)



10. Solve the equation 110 x =1 **mod** 273. (10%)