國立彰化師範大學 102 學年度碩士班招生考試試題

系所：資訊工程學系　　　　科目：離散數學及線性代數

☆☆請在答案紙上作答☆☆　　　　

(10% each)

1. Determine <u>all values</u> of s so that $\boldsymbol{A}$ is ***nonsingular***, where $\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & s & 2 \end{bmatrix}$.

2. Compute the value *det(**B**)* of the matrix **B**, where $\boldsymbol{B}=\begin{bmatrix} 1 & 2 & -3 & 4 \\ -1 & 2 & 1 & 3 \\ 3 & 0 & 0 & -3 \\ 2 & 0 & 2 & 0 \end{bmatrix}$.

3. Find the ***eigenvalues*** and respect ***eigenvectors*** of the matrix **C**, where $\boldsymbol{C} = \begin{bmatrix} 0 & -2 & 1 \\ 1 & 3 & -1 \\ 0 & 0 & 1 \end{bmatrix}$.

4. Let $\boldsymbol{V}= \boldsymbol{R}^3$ and $\boldsymbol{S}=\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ be an ordered basis for **V**, where $\boldsymbol{v}_1=[1\ 1\ 0]^T$, $\boldsymbol{v}_2=[2\ 0\ 1]^T$, and $\boldsymbol{v}_3= [0\ 1\ 2]^T$. Given vector $\boldsymbol{u}=[1\ 1\ -5]^T$, find the **coordinates vector** of **u** relative to the ordered basis **S**.

5. Determine an ***LU-factorization*** of matrix **E**, where $\boldsymbol{E}=\begin{bmatrix} 6 & -2 & -4 & 4 \\ 3 & -3 & -6 & 1 \\ -12 & 8 & 21 & -8 \\ -6 & 0 & -10 & 7 \end{bmatrix}$.

6. Find the two smallest positive solutions to the *system of congruences*.
   $x \equiv 1 \bmod 4$, $x \equiv 2 \bmod 5$, $x \equiv 3 \bmod 7$

7. Construct a ***truth table*** for the following *compound propositions*: $(p \vee q) \rightarrow (p \oplus q)$

8. Prove that if n is a positive integer, then 133 divides $11^{n+1} + 12^{2n-1}$.

9. Solve these ***recurrence relations*** with the initial conditions given.
   $a_n = -6a_{n-1} - 9a_{n-2}$, $a_0 = 3$, $a_1 = -3$
   $a_n = a_{n-1} + 2n + 3$, $a_0 = 4$

10. **Prove or disprove**: if A, B, and C are sets, then $A - (B \cap C) = (A - B) \cap (A - C)$.