國立彰化師範大學 100 學年度碩士班招生考試試題

系所：資訊工程學系　　　　　　　　　　　　科目：離散數學及線性代數

☆☆請在答案紙上作答☆☆　　　　　　　　　　　　共 2 頁，第 1 頁

1. Given the *symmetric matrix* $\boldsymbol{A}$, find $\boldsymbol{e}^{\boldsymbol{A}}$. (10%)

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 0 & -2 & 0 \\ -2 & 0 & 3 \end{bmatrix}$$

2. Let W be the subspace of $\boldsymbol{R}^4$ with basis $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$, where $\mathbf{u}_1 = (1, -2, 0, 1)$, $\mathbf{u}_2 = (-1, 0, 0, -1)$, and $\mathbf{u}_3 = (1, 1, 0, 0)$. Use the **Gram-Schmidt process** to transform S to an *orthonormal basis* for W. (10%)

3. Find the *inverse* of the given matrix $\boldsymbol{B}$. (10%)

$$B = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 4 & 1 & 2 & 1 \\ 2 & -1 & 3 & 1 \\ 4 & 2 & 1 & -5 \end{bmatrix}$$

4. Evaluate the *determinant* (10%)

$$\left\| \begin{bmatrix} 1 & 1 & 2 & -1 \\ 0 & 1 & 0 & 3 \\ -1 & 2 & -3 & 4 \\ 0 & 5 & 0 & -2 \end{bmatrix} \right\|$$

5. Given the matrix C, find the *rank(C)*, *nullity (C)*, and show that *rank(C)+nullity(C) =5*. (10%)

$$C = \begin{bmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & -1 & 0 & 0 & 2 \\ 2 & 1 & 6 & 0 & 1 \end{bmatrix}$$

6. Use rules of inference to show if $\forall x\,(P(x) \to (Q(x) \wedge S(x)))$ and $\forall x\,(P(x) \wedge R(x))$ are true, then $\forall x\,(R(x) \wedge S(x))$ is true. (10%)

7. Use mathematics induction to prove the statement: $11^n - 6$ is divisible by 5, for n =1, 2,... (10%)

8. Solve the following recurrence relation:
   (a) $9a_n = 6a_{n-1} - a_{n-2}$, for $n \geq 2$ with initial condition a_0=6, a_1=5. (5%)
   (b) $\sqrt{a_n} = \sqrt{a_{n-1}} + 2\sqrt{a_{n-2}}$, for $n \geq 2$ with initial condition $a_0 = a_1$=1. (5%)

9. Eight coins are identical in appearance, but one coin is heavier than the others, which all weigh the same. Draw an optimal decision tree to identify the bad coin using only a pan balance. (10%)

[考生注意!!第二面尚有試題]

10. Consider the transport network shown in the following figure.

(a) Find the maximum flow by using the labeling procedure. (6%)

(b) Let *G(V, E)* be the network, *P* be the subset of *V*, and $\overline{P} = V \setminus P$. A *cut* of network *G* is a subset of *E*. The cut partitions the vertices of the network into the two sets *P* and its complement $\overline{P}$. The *capacity of a cut* is denoted as $cut(P, \overline{P})$. Determine the corresponding minimal $cut(P, \overline{P})$. (4%)