

國立彰化師範大學 99 學年度碩士班招生考試試題

系所：資訊工程學系

科目：離散數學及線性代數

☆☆請在答案紙上作答☆☆

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1. Construct a *nondeterministic finite-state automaton* that recognizes the *regular set* $1^* \cup 01$. (10%)
2. The chromatic number of a graph is the least number of colors needed for a coloring of this graph. What is the *chromatic number* of the graph (a) complete graph K_n , (b) complete bipartite graph $K_{m,n}$, (c) cycle C_n , (d) wheel W_n , (e) n -Cubes Q_n . (10%)
3. Find $3^{644} \bmod 645$. (10%)
4. Show that there exist *irrational* numbers x and y such that x^y is *rational*. (10%)
5. What is the solution of the *recurrence relation* $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$ and $a_1 = 7$? (10%)
6. Find the *transition matrix* corresponding to the change of basis from $[\mathbf{v}_1, \mathbf{v}_2]$ to $[\mathbf{u}_1, \mathbf{u}_2]$, where $\mathbf{v}_1 = [5, 2]^T$, $\mathbf{v}_2 = [7, 3]^T$ and $\mathbf{u}_1 = [3, 2]^T$, $\mathbf{u}_2 = [1, 1]^T$. (10%)
7. Let $A = \begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{bmatrix}$, find an *orthonormal basis* for the *column space* of A . (10%)
8. Solve the linear system whose augmented matrix is $\left[\begin{array}{ccc|c} i & 2 & 1-i & 1-2i \\ 0 & 2i & 2+i & -2+i \\ 0 & -i & 1 & -1-i \end{array} \right]$. (10%)
9. Consider two 2×2 matrices, A and B . We are told that $B^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, and $(AB)^{-1} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$, Find A . (10%)
10. Find an *LU factorization* of $A = \begin{bmatrix} 6 & -2 & -4 & 4 \\ 3 & -3 & -6 & 1 \\ -12 & 8 & 21 & -8 \\ -6 & 0 & -10 & 7 \end{bmatrix}$. (10%)