國立彰化師範大學 98 學年度<u>碩士班</u>招生考試試題

系所：資訊工程學系　　　　　　　　　　　　　科目：離散數學及線性代數

1. (a) Solve the linear system $\begin{cases} 2x+8y+4z=2 \\ 2x+5y+z=5 \\ 4x+10y-z=1 \end{cases}$. (5%)

   (b) Consider two 2×2 matrices, A and B. We are told that $B^{-1}=\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, and $(AB)^{-1}=\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$. Find A. (5%)

2. (a) Find the *kernel* of the linear transformation T($\mathbf{x}$)=$A\mathbf{x}$ from R^3 to R^2, where $A=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$.(5%)

   (b) For which values of the constant k do the following vectors form a *basis* of R^3?

   $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ k \\ k^2 \end{bmatrix}$.(5%)

3. (a) Is matrix $A=\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ similar to $B=\begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix}$? Justify your answer. (5%)

   (b) Find the *angle* between the vectors $\mathbf{x}=\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ and $\mathbf{y}=\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.(5%)

4. (a) Find the *inverse* of the matrix $\begin{bmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{bmatrix}$.(5%)

   (b) Find the *determinant* of the matrix A=$\begin{bmatrix} 1 & 0 & 1 & 0 & 2 \\ -9 & 0 & 3 & 0 & 0 \\ 1 & 2 & 3 & 4 & 5 \\ 9 & 2 & 2 & 0 & 0 \\ 5 & 0 & 0 & 0 & 3 \end{bmatrix}$.(5%)

5. (a) Consider the matrix $A=\begin{bmatrix} 1/2 & 3/4 \\ 1/2 & 1/4 \end{bmatrix}$, find a matrix S that *diagonalizes A*. (5%)

    (b) find $\lim_{k\to\infty} A^k$. *(5%)*

6. (a) What is the *chromatic number* of *graph Wheel W_n?* (5%)

    (b) How many numbers must be selected from the set {1, 3, 5, 7, 9, 11, 13, 15} to guarantee that at least one pair of these numbers add up to 16? (5%)

7. (a) Find the *transitive closure* of the relation **{(a, b), (b, c), (d, a)}**. (5%)

    (b) **Decrypt** the message encrypted using the *Caesar cipher* "**HDWGLPVXP**". (5%)

8. (a) If there are 2187 functions $f:A\to B$ can be defined on sets A and B, and $|B|=3$, what is $|A|$? (5%)

    (b) Find the solutions of the *linear congruence $3x \equiv 4 \bmod 7$*. (5%)

9. (a) Let $\delta(x, y)$ be the *Hamming Distance* of the x and y, compute $\delta(110110,000101)+\delta(001100,010110)$. (5%)

    (b) Let Z^+ be the set of positive integers, and let $R=\{(a,b)\in Z^+\times Z^+ | a$ divides $b\}$. Is R *symmetric*, *asymmetric*, or *antisymmetric*? (5%)

10. (a) Choose four cards at random from a standard 52-card deck. What is the probability that four kings will be chosen? (5%)

    (b) Let *GCD(x,y)* be the *greatest common divisor* of positive integers x and y. Find *GCD(123456789 ,987654321)*. (5%)