國立彰化師範大學105學年度碩士班招生考試試題

系所： 資訊工程學系(選考辛)　　　　科目： 線性代數與機率
電信工程學研究所(選考戊)

☆☆請在答案紙上作答☆☆　　　　

1. True or False? Determine whether each statement is true or false. (15%)
   (1) If the matrices A, B, and C satisfy $AB = AC$, then $B = C$.
   (2) If a subset S spans a vector space V, the every vector in V can be write as a linear combination of the vectors in S.
   (3) A set S of vectors in an inner product space V is orthonormal if every vector is a unit vector and each pair of vectors in S is orthogonal.
   (4) Any linear function of the form $f(x) = ax + b$ is a linear transformation from R to R.
   (5) If A is an $n \times n$ matrix with an eigenvalue λ and corresponding eigenvector $\mathbf{x}$, then every nonzero scalar multiple of $\mathbf{x}$ is also an eigenvector of A.

2. For the system of linear equations,
   $x + y = 0$
   $x - y - z = 0$
   $y + z = 1$
   Please use the inverse of coefficient matrix to solve the system. (10%)

3. Given the basis $[(1, 2, -2)^{\mathbf{T}}, (4, 3, 2)^{\mathbf{T}}, (1, 2, 1)^{\mathbf{T}}]$ for $\mathbf{R}^3$, use the Gram-Schmidt process to obtain an orthonormal basis. (10%)

4. Let $\mathbf{B}=[\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}]=[(1, 3, 1)^T, (2, 1, 4)^T, (3, -2, 3)^T]$. What is the coordinate vector of $\mathbf{w}=(3, 17, 13)^T$ with respect to the basis $\mathbf{B}=[\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}]$. (5%)

5. For the matrix $\mathbf{A}$, find (1) $\mathbf{det(A)}$ and (2) $\mathbf{A}^{-1}$. (10%)

$$\mathbf{A}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 3 & -2 & 2 & 1 \\ 0 & 15 & 0 & 1 \\ 5 & 5 & 5 & 5 \end{bmatrix}$$

6. True or False? Determine whether each statement is true or false. (15%)
   (1) The events E, F and G are mutually exclusive, then $P(E \cup F \cup G) = P(E) + P(F) + P(G)$.
   (2) Three events E, F and G are independent if $P(EFG) = P(E)P(F)P(G)$.
   (3) Let X be a random variable and $Y = aX + b$, where a and b are constants. Then, the variance $\mathrm{Var}(Y) = a^2\ \mathrm{Var}(X)$.
   (4) The exponential random variables are the unique memoryless random variable.
   (5) For two random variables X and Y, X and Y are independent if the covariance $\mathrm{Cov}(X, Y) = 0$.

7. A fair coin that is flipped 100 times. Let X be the number of heads that appear. Find the probability $P(X < 10)$. (10%)

8. Let Y represent the amount of time (in minutes) you must wait for a friend whom you agree to meet at 12 noon; assume the *cumulative distribution function* for Y is $F_Y(y) = \begin{cases} 1-\dfrac{1}{t^3}, & \text{for } t>1 \\ 0, & \text{otherwise} \end{cases}$.
   (1) Evaluate the probability that you must wait at least 2 minutes for your friend. (5%)
   (2) Find the pdf for Y. (5%)

9. If the cumulative distribution function of X is given by following formula,

$$F(b) = \begin{cases} 0 & b < 0 \\ 1/2 & 0 \le b < 1 \\ 3/5 & 1 \le b < 2 \\ 4/5 & 2 \le b < 3 \\ 9/10 & 3 \le b < 3.5 \\ 1 & b \ge 3.5 \end{cases}$$

   (1) calculate the probability mass function of X, (5%)
   (2) what is the expected value of random variable X. (5%)

10. Show that if $P(\mathbf{A}|\mathbf{B}) = 1$, then $P(\mathbf{B}^c|\mathbf{A}^c) = 1$, (where $\mathbf{B}^c$ is the *complement* of set **B**) (5%)