

# 國立彰化師範大學105學年度碩士班招生考試試題

系所： 車輛科技研究所(選考甲)、機電工程學系(選考甲)、電機工程學系(選考甲)、電子工程學系(甲組選考甲)、電子工程學系(乙組選考甲)、電信工程學研究所(選考甲)、資訊工程學系(選考戊)、資訊工程學系積體電路設計碩士班(選考甲) 科目：工程數學

☆☆請在答案紙上作答☆☆

共 1 頁，第 1 頁

1. Let  $f(x)=1$  for  $0 < x < 2$ . Suppose that  $f(x)$  is represented by the Fourier series of the form

$$\sum_{n=1,3,5,\dots}^{\infty} B_n \sin\left(\frac{n\pi}{2}x\right).$$

- (a) What's the period of this Fourier series? (5%)
- (b) Find the coefficients  $B_n$ . (10%)
- (c) At  $x=0, 1, -1, 2$ , what value does this Fourier series converge to? (10%)

(d) Find  $\sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2}$  (10%)

2. Solve the given initial-value problem. (15%)

$$y''' - 2y'' + y' = xe^{-x} + 5,$$
$$y(0) = 2, y'(0) = 2, y''(0) = -1$$

3. Consider the following vector equation (or called a state equation)

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t), \text{ with } \mathbf{A} = \begin{bmatrix} 0 & 1 \\ -4 & -4 \end{bmatrix},$$

where  $t$  is the time starting from zero,  $(\cdot) = \frac{d}{dt}$ , and  $\mathbf{x}(t) = (x_1(t), x_2(t))^T$ ; the superscript  $T$  denotes

the transpose of the vector  $\mathbf{x}$ . Please answer the following questions :

- (a) Prove that  $\mathbf{x}(t) = \mathbf{c}e^{\mathbf{A}t}$ ,  $\mathbf{c}$  being an eigenvector, is the solution of the vector equation shown above. (10%)
- (b) Based on part (a), find the initial condition of the vector equation. (10%)
- (c) By taking the Laplace transform to the vector equation, prove that  $e^{\mathbf{A}t} = L^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}]$ , where the symbol  $L^{-1}$  indicates the inverse of the Laplace transform, and  $\mathbf{I}$  presents the unit vector. (10%)
- (d) Find  $L^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}]$  in time domain. (20%)

**Note :** The inverse of the Laplace transform that you need is  $L^{-1}\left[\frac{n}{(s-a)^{n+1}}\right] = t^n e^{at}$ ,  $n$  being a positive integer.