國立彰化師範大學103學年度碩士班招生考試試題

系所： 車輛科技研究所(選考 1)、
電子工程學系(甲組選考甲、乙組選考甲)、
資訊工程學系積體電路設計碩士班(選考甲)、
電信工程學研究所(選考甲)　　　　　　　　　　　科目：　工程數學

☆☆請在答案紙上作答☆☆　　　　　　　　　　　　　　　　共 3 頁，第 1 頁

重要提醒:

考生可任選 **10** 題作答並在答案紙上標註確認之，

否則以答案紙上答題順序前 **10** 題計分

1. Solve the system of linear first-order differential equations. (10%)

$$\begin{cases} \dfrac{dx_1(t)}{dt} = -\dfrac{2}{25} x_1(t) + \dfrac{1}{50} x_2(t) \\ \dfrac{dx_2(t)}{dt} = \dfrac{2}{25} x_1(t) - \dfrac{2}{25} x_2(t) \end{cases}$$

2. Solve the differential equation $y'' + 2y' + y = \cos x - xe^{-x}$. (10%)

3. Solve the differential equation $x^3 y''' + xy' - y = x(\ln x)$. (10%)

4. Use the Laplace transform to solve the given initial-value problem. (10%)

$$y'' + 16y = f(t),\ y'(0) = 1;\ y(0) = 0$$

$$f(t) = \begin{cases} \cos 4t, & 0 \le t < \pi \\ 0, & t \ge \pi \end{cases}$$

5. A period function $f(x) = f(x + 2\pi)$ can be represented by a trigonometric series

   $f(x) = a_0 + \sum\limits_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$. Please derive the formula for the coefficients a_0, a_n,

   and b_n. (10%)

6. Find the eigenvalues and the corresponding eigenfunctions of the Strum-Liouville problem. (10%)

$$y'' + \lambda y = 0,\ y(0) = y(1),\ y'(0) = y'(1)$$

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7. Find the half-range Fourier cosine expansion of $f(x) = \begin{cases} \dfrac{2k}{L}x, & 0 < x < \dfrac{L}{2} \\ \dfrac{2k}{L}(L-x), & \dfrac{L}{2} < x < L \end{cases}$. Sketch its periodic extension from *-2L* to *2L*. (10%)

8. Find the complex Fourier series of $f(x) = e^x$ if $-\pi < x < \pi$ and $f(x+2\pi) = f(x)$. (10%)

9. Consider the Strum-Liouville boundary value problem $y'' + \lambda y = 0,\ y(0) = 0,\ y'(L) = 0$. Show that if $\phi_m(x)$ and $\phi_n(x)$ are eigenfunctions corresponding to the eigenvalues λ_m and λ_n, respectively, with $\lambda_m \neq \lambda_n$, then $\int_0^L \phi_m(x)\phi_n(x)\,dx = 0$. (10%)

10. A periodic function $f(t)$ is defined as $f(t) = \begin{cases} 0, & -2 < t < -1 \\ 1, & -1 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$ with the period of $T = 4$. As $T = \infty$, please find the Fourier integral representation of the function. (10%)

11. Let X be the number of times that a fair coin that is flipped 100 times lands on heads. Use the normal approximation to find the probability that X > 70. (10%)

12. Consider the random process with sample functions $x(t) = \cos(2\pi f_0 t + \Phi)$, where f_0 is a constant and Φ is a random variable uniformly distributed over $[-\pi,\ \pi]$. Please find the probability density function of Φ and, computed as statistical averages, find the first and second moments of $x(t)$. (10%)

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13. For the system of linear equations, please use the Gauss-Jordan elimination to solve the system. (10%)

$$-x + 2y + z = 1$$
$$2x + 3y + z = -2$$
$$5x + 4y + 2z = 4$$

14. Following the Problem13, please use the inverse of coefficient matrix to solve the system. (10%)

15. Let $f(x) = cos(mx)$ and $g(x) = cos(nx)$ be functions in the inner product space $C[0,\ 2\pi]$ with the standard inner product, where m and n are positive integers. Find the inner product $\langle f, g \rangle$. (10%)