國立彰化師範大學 99 學年度碩士班招生考試試題

系所：電子工程學系　　　組別：甲組、乙組(選考甲)　　　科目：工程數學

☆☆請在答案紙上作答☆☆　　　　　　　　　　　　　

1. (30%) Solve the following equations.

   (a) $(2x^2y+5x+3)dy+(2xy^2+5y+1)dx=0$

   (b) $y'''-3y'+2y=8e^x \qquad y(0)=2,\ \ y'(0)=1,\ \ y''(0)=9$

   (c) $x^2y''-2xy'+2y=x$

2. (20%) In (a), find the inverse Laplace transforms. In (b), find the Laplace transforms.

   (a) $\dfrac{e^{-ks}}{(s-3)^2}$

   (b) $2t^2e^{-at}$

3. (13%) Find the eigenfunction expansion of the given function $f(x)=\begin{cases}-1, & 0\le x\le 2\\ 1, & 2<x\le 4\end{cases}$ by using the eigenfunctions of the Strum-Liouville problem $y''+\lambda y=0,\ y'(0)=y(4)=0$. Determine what the eigenfunction expansion converges to on the interval.

4. (15%) (a) Prove the following form of Parseval theorem $\int_{-\infty}^{\infty}|f(t)|^2\,dt=\dfrac{1}{2\pi}\int_{-\infty}^{\infty}|F(\omega)|^2\,d\omega$ where $F(\omega)$ is the Fourier transform of $f(t)$. (b) Find the Fourier transform of $f(t)=\begin{cases}a, & |t|\le a\\ 0, & |t|>a\end{cases}$.

   (c) Use the Parseval theorem to evaluate the integral $\int_{-\infty}^{\infty}\left(\dfrac{sin(\omega a)}{\omega a}\right)^2 d\omega$.

5. (12%) Let $\mathbf{r}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ and $r=|\mathbf{r}|$. If $\mathbf{a}$ is an arbitrary constant vector, (a) what is $(\mathbf{a}\cdot\nabla)\mathbf{r}$ and $(\mathbf{a}\times\nabla)\cdot\mathbf{r}$? (b) Prove that $\nabla\times\left[\dfrac{1}{r}(\mathbf{a}\times\mathbf{r})\right]=\dfrac{1}{r}\mathbf{a}+\dfrac{\mathbf{a}\cdot\mathbf{r}}{r^3}\mathbf{r}$.

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6. (10%) Consider the differential equation $\dfrac{dy}{dx}=\dfrac{ax+by+c}{Ax+By+C}$ with $aB\neq bA$. (a) Solve the differential equation $\dfrac{dy}{dx}=\dfrac{x-y-1}{x+y+3}$ by finding h and k so that the substitutions $x=u+h$, $y=v+k$ transform it into the homogeneous equation $\dfrac{dv}{du}=\dfrac{u-v}{u+v}$. (b) Use this method to solve the differential equation $\dfrac{dy}{dx}=\dfrac{x-y-1}{x+y+3}$.