

# 國立彰化師範大學 98 學年度碩士班招生考試試題

系所：電子工程學系

組別：甲、乙組

科目：工程數學

☆☆請在答案紙上作答☆☆

共 2 頁，第 1 頁

1. (15%) Calculate the circulation of vector function  $\mathbf{F} = (x - y)\mathbf{i} + x^2y\mathbf{j} + xz\mathbf{k}$  counterclockwise about the circle  $x^2 + y^2 = 1$ . Here  $a$  is a positive constant.
2. (15%) Use the Fourier Transform to solve  $y'' + 6y' + 5y = \delta(t - 3)$ .
3. (20%) Legendre's equation is  $(1 - t^2)y'' - 2ty' + \mu(\mu + 1)y = 0$ . This equation has a power series solution of the form  $y(t) = \sum_{n=0}^{\infty} a_n t^n$  that is guaranteed to be absolutely convergent in the interval  $-1 < t < 1$ .
  - (a) Find the recurrence relation for the coefficients of the power series.
  - (b) Argue, when  $\mu = N$  is a nonnegative integer, that Legendre's equation has a polynomial solution,  $P_N(t)$ .
  - (c) Use the recurrence relation and the requirement that  $P_N(1) = 1$  to determine the first three Legendre polynomials,  $P_1(t)$ ,  $P_2(t)$ ,  $P_3(t)$ .
4. (20%) Consider the Sturm-Liouville boundary value problem  $y'' + 2y' + (1 + \lambda)y = 0$ ,  
 $y(0) = y(1) = 0$ .
  - (a) Classify the Sturm-Liouville boundary value problem as regular, periodic or singular; state the relevant interval.
  - (b) Find the eigenvalues  $\lambda_n$  and eigenfunctions  $\phi_n(x)$ .
  - (c) Find the eigenfunction expansion of the function  $f(x) = \begin{cases} 0, & 0 \leq x \leq 0.5 \\ 1, & 0.5 < x \leq 1 \end{cases}$  in the eigenfunctions of the Sturm-Liouville problem.
  - (d) Determine what the eigenfunction expansion converges to on the interval.

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5. (15%) Consider the initial value problem  $y'' + by' + cy = f(x)$ ,  $0 < t < \infty$ ,  $y(0) = y_a$  and  $y'(0) = y_b$ . The input function  $f(t) = e^{-t}$  and the Laplace transform of the output function  $y(t)$  is  $Y(s) = \frac{s^2 + s + 1}{(s+1)(s^2 + 4)}$ . Determine the constants of  $b$ ,  $c$ ,  $y_a$ ,  $y_b$ .
6. (15%) Find the Cauchy principal value of  $\int_{-\infty}^{+\infty} \frac{\cos(3x)}{x-1} dx$ .