國立彰化師範大學 99 學年度碩士班招生考試試題

系所：統計資訊研究所　　　　科目：基礎數學

☆☆請在答案紙上作答☆☆　　　　

1. (a) A matrix $C=(c_{ij})_{1\le i,j\le n}$ is called orthogonal if it satisfies the following two conditions

$$\sum_{j=1}^{n} c_{ij}^2 = 1 \quad and \quad \sum_{j=1}^{n} c_{ij} c_{i'j} = 0,\ i, i' = 1,2,\cdots,n,\ i \ne i'.$$

Show that if $x_i = \sum_{j=1}^{n} c_{ji} y_j,\ i = 1,2,\cdots,n,$ then

$$\sum_{i=1}^{n} x_i^2 = \sum_{j=1}^{n} y_j^2.$$ (10 points)

(b) Define

$$c_{1j} = 1/\sqrt{n},\ \ j = 1,2,\cdots,n,$$

$$c_{ij} = \begin{cases} 1/\sqrt{i(i-1)}, & for\ i = 2,\cdots,n\ \ and\ \ j = 1,\cdots,i-1; \\ 0, & for\ \ j = i+1,\cdots,n; \end{cases}$$

$$c_{ii} = -(i-1)/\sqrt{i(i-1)},\ \ i = 2,\cdots,n.$$

Check if $C=(c_{ij})_{1\le i,j\le n}$ is an orthogonal matrix. (10 points)

2. Let $D = \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0.3 & 0.5 & 0.2 \\ 0.5 & 0.4 & 0.1 \end{bmatrix}$. Find the limit of the n-th matrix product $\lim_{n\to\infty} D^n$. (15 points)

3. Calculate the following integral.(15 points)

$$f(t) = \int_0^\infty \frac{1}{\sqrt{2\pi r}\,\Gamma(r/2)2^{r/2}} u^{\frac{r-1}{2}} \exp\left[-\frac{u}{2}\left(1+\frac{t^2}{r}\right)\right] du,\ t \in \Re,$$

where $\Gamma(x)$ is the gamma function and $r \ge 1$ *is an* integer.

4. Transform the basis {[1,0,1],[2,2,2],[2,3,1]} for $\Re^3$ into an orthogonal basis for $\Re^3$ using the Gram-Schmidt process. (15 points)

5. (20 points)
   (a) Let V be a real vector space and $v \in V$ *and* $r \in \Re$. Show that $rv = 0$ implies either $r = 0$ *or* $v = 0$.
   (b) Let $T : U \to U$ be a linear transformation with the property that $T(T(v)) = T(v) + 5v$ for all $v \in V$. Show that T is one-to-one.

6. Using $\int_0^\infty \frac{\sin x}{x} dx = \pi / 2$ to find the following integrals. (15 points)

   (a) $\int_0^\infty \frac{1 - \cos x}{x^2} dx$,　　(b) $\int_0^\infty \left(\frac{\sin x}{x} \right)^2 dx$.