國立彰化師範大學 98 學年度<u>碩</u>士班招生考試試題

1、Let U be the subspace of $\Re_3[x]$ generated by $1+2x+x^3$ and $1-x-x^2$. Let V be the subspace of $\Re_3[x]$ generated by $x+x^2-3x^3$ and $2+2x-2x^3$.
(a) Find the dimension of $U+V$. (10%)
(b) Give a basis for $U \cap V$. (10%)

2、Define $H = X\left(X^T X\right)^{-1} X^T$, where X is an $n \times k$ dimensional matrix with $n > k$ and X^T is the transpose of X. Please show that the ranks of H and $I_n - H$ are k and $n-k$, respectively. (20%)

3、Let $A = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$. Find A^{10} and $\lim_{n \to \infty}(A^{-1})^n$. (20%)

4、(a) State without proof a version of the Fundamental Theorem of Calculus. (10%)
(b) Find $\dfrac{d}{dx}\displaystyle\int_{\frac{\pi}{2}}^{x^3} \cos t\, dt$. (10%)

5、Define $\Gamma(\alpha) = \displaystyle\int_0^\infty x^{\alpha-1}e^{-x}dx$ for $\alpha > 0$.

(a) Prove $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$. (10%)
(b) Find the value of $\Gamma(5)$. (10%)