

# 國立彰化師範大學104學年度碩士班招生考試試題

系所： 統計資訊研究所

科目： 微積分

☆☆請在答案紙上作答☆☆

共1頁，第1頁

1. Find the limit  $\lim_{x \rightarrow 0} (1 + \sin 3x)^{\frac{1}{x}}$ . (15%)

2. Solve the integral equation  $y(x) = 2 + \int_0^x y(t) dt$ , for all  $x \in R$ . (15%)

3. Evaluate the following integrals (20%)

(a)  $\int_0^1 \frac{1}{2+e^x} dx$ .

(b)  $\int_0^{\pi/2} \frac{1}{2+\cos x} dx$ .

4. Let  $f(x) = \frac{x^3}{x^2+1}$ , for  $x \in R$  (15%)

(a) Show that  $f(x)$  is one-to-one. (So it has an inverse  $y = f^{-1}(x)$ .)

(b) Find the slope of the function  $y = f^{-1}(x)$  at the point  $x = \frac{1}{2}$ .

5. Evaluate the integral  $\int_0^4 \int_{\sqrt{y}}^2 e^{-x^3} dx dy$ . (15%)

6. Find all real numbers  $p$  for which the series  $\sum_{n=2}^{\infty} \frac{1}{n (\ln n)^p}$  is convergent.

Explain your answer. (20%)