

國立彰化師範大學 101 學年度碩士班招生考試試題

系所： 統計資訊研究所

科目： 微積分

☆☆請在答案卷上作答☆☆

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(共 5 題，每題 20 分，共 100 分)

1. Find the limit

$$\lim_{h \rightarrow 0} \frac{1}{h} \int_2^{2+5h} \frac{1}{\ln t} dt.$$

2. Prove that $|e^{-x^2} - e^{-y^2}| \leq M|x - y|$ for some $M < 1$.

3. Prove that $f(x) = \ln \frac{1+x}{1-x} = 2 \sum_{k=0}^{\infty} \frac{x^{2k+1}}{2k+1}$, for $-1 < x < 1$. Explain the convergence of the infinite series.

4. Test for convergence or divergence:

(a) $\sum_{n=2}^{\infty} \frac{1}{n \log n}$

(b) $\sum_{n=30}^{\infty} \frac{1}{n \log n [\log(\log(n))]}$

(c) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{n}$

5. $f(x) = \frac{1}{\sqrt{\nu} B(\frac{1}{2}, \frac{\nu}{2})} \left(1 + \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}},$

where B is the beta function. Show that as $\nu \rightarrow \infty$, $f(t) \rightarrow \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$.