

# 國立彰化師範大學 102 學年度碩士班招生考試試題

系所：電信工程學研究所

科目：工程數學

☆☆請在答案紙上作答☆☆

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1. A random variable  $X$  has the probability density function  $f_X(x) = ae^{-b|x|}$ 
  - (a) Find the relationship between  $a$  and  $b$ . (5%)
  - (b) Calculate the expectation of  $X$ . (5%)
  - (c) Calculate the variance of  $X$ . (5%)
2. For a Gaussian random variable  $X$  with mean  $m_X = 1$  and variance  $\sigma_X^2 = 0.1$ , express the probability  $P(X < 0)$  in term of the  $Q$  function. (10%)
3. Find the inverse of the matrix  $A = \begin{bmatrix} 1 & -2 & -1 & -2 \\ 3 & -5 & -2 & -3 \\ 2 & -5 & -2 & -5 \\ -1 & 4 & 4 & 9 \end{bmatrix}$ . (10%)
4. Let  $f(x) = 1$  and  $g(x) = x$  be functions in the inner product space  $C[-1, 1]$ . Find (a) the inner product  $\langle f, g \rangle$ , (b) the distance  $d(f, g)$ , and (c) the orthogonal projection of  $f$  onto  $g$ . (15%)
5. A function is defined as  $f(x) = x^2$ ,  $0 \leq x < 1$ .
  - (a) Find the Fourier coefficients in the half-range sine expansion of the function. (10%)
  - (b) Using the results of (a), calculate the summation of the infinite series  $1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \dots$ . (10%)
6. Find the general solution for  $y'' - 4y' + 4y = x^3e^{2x} + xe^{2x}$ . (10%)
7. Find the eigenvalues and the corresponding eigenfunctions of the boundary-value problem. (10%)
$$y'' + \lambda y = 0 \quad y(-1) = y(1) = 0 \quad y'(-1) = y'(1) = 0$$
8. Use the Laplace transform to solve the integral-differential equation. (10%)
$$y'' + y' - 4 \int_0^t y(\tau) \sin(t - \tau) d\tau = e^{-2t}, \quad y(0) = 1, \quad y'(0) = 0$$