國立彰化師範大學 100 學年度碩士班招生考試試題

系所：電信工程學研究所　　　　　　　　　　科目：工程數學

☆☆請在答案紙上作答☆☆　　　　　　　　　　

1. (20%) Given a periodic function $f(x)=\begin{cases}0, & -L<x<0\\ L, & 0<x<L\end{cases}$ with $f(x+2L)=f(x)$ for all x.

   (a) Please find the Fourier series for this periodic function.

   (b) From the Fourier series of (a), please deduce the value of the sum of the series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$

2. (15%) Consider the nonhomogeneous differential equation $y'''+ay''+by'+cy=g(t)$. The general solution of this differential equation is given as $y(t)=c_1+c_2t+c_3e^{2t}+4\sin 2t$. Use this information to determine the constants a, b, c, and the function $g(t)$.

3. (15%) Use Laplace transforms to solve the following systems of differential equations.

$$\begin{aligned}&4y_1'+y_2'-2y_3'=0\\ &-2y_1'+y_3'=1\\ &2y_2'-4y_3'=-16t\\ &y_1(0)=2 \quad y_2(0)=0 \quad y_3(0)=0\end{aligned}$$

4. (10%) The determinant is known as $det\begin{bmatrix}1&1&1&1\\1&2&3&4\\1&3&6&10\\1&4&10&20\end{bmatrix}=1$, find the $det\begin{bmatrix}1&1&1&1\\1&2&3&4\\1&3&6&10\\1&4&10&19\end{bmatrix}=?$

5. (10%) Find the least squares approximation of the system $A\underline{x}=\underline{b}$ where $A=\begin{bmatrix}1&1\\4&4\\-2&2\end{bmatrix}$ and $\underline{b}=\begin{bmatrix}1\\1\\-2\end{bmatrix}$ (note: to find the least squares solution $\underline{\hat{x}}$ and $\underline{p}$, where $A\underline{\hat{x}}=\underline{p}$, and $\underline{p}$ is the least square approximation to $\underline{b}$.)

6. (10%) Find the eigenvalues and eigenvectors of $\boldsymbol{A}$ and $\boldsymbol{A}^2$ and $\boldsymbol{A}+4\boldsymbol{I}$ ($\boldsymbol{I}$ is the identity matrix):.

$$\boldsymbol{A} = \begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$$

7. (10%) Find the *pdf*, $f_{\mathbf{Z}}(z)$, for $\mathbf{Z} = \mathbf{X}^2$ with the given $f_{\mathbf{X}}(x) = \dfrac{x}{\alpha^2} e^{-x^2/2\alpha^2}$, $x > 0$, $\alpha > 0$.

8. (10%) If $\mathbf{X}$ is a Gaussian random variable with mean μ and variance σ^2, show that $\mathbf{Y} = a\mathbf{X}+b$, where a and b are real constants, is also a Gaussian random variable. Find $\boldsymbol{E}(\mathbf{Y})$ and $\boldsymbol{Var}(\mathbf{Y})$.