國立彰化師範大學 99 學年度碩士班招生考試試題

系所：電信工程學研究所　　　　科目：工程數學

☆☆請在答案紙上作答☆☆　　　　

1. (12%) Please diagonalize matrix $\boldsymbol{B}=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$ using the corresponding eigenvector matrix $\mathbf{S}$ and eigenvalue matrix $\boldsymbol{\Lambda}$, and compute $\mathbf{S}\boldsymbol{\Lambda}^k\mathbf{S}^{-1}$ to prove that $B^k=\frac{1}{2}\begin{bmatrix}3^k+1 & 3^k-1\\ 3^k-1 & 3^k+1\end{bmatrix}$

2. (12%) A linear transformation $\mathbf{T}$ from $\boldsymbol{V}$ vector space to $\boldsymbol{W}$ vector space is represented by a matrix $\mathbf{A}$ after the bases are chosen for $\boldsymbol{V}$ and $\boldsymbol{W}$. With vector bases $\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}$ and $\boldsymbol{w_1}, \boldsymbol{w_2}, \boldsymbol{w_3}$, suppose $\mathbf{T}(\boldsymbol{v_1})=\boldsymbol{w_2}$ and $\mathbf{T}(\boldsymbol{v_2})=\mathbf{T}(\boldsymbol{v_3})=\boldsymbol{w_1}-\boldsymbol{w_3}$. Find (a) the matrix $\mathbf{A}$, (b) $\mathbf{T}(\boldsymbol{v_1}+\boldsymbol{v_2}+\boldsymbol{v_3})$, (c) the solutions of $\mathbf{T}(\boldsymbol{v})=\mathbf{0}$, and (d) all the solutions to $\mathbf{T}(\boldsymbol{v})=\boldsymbol{w_2}$.

3. (15%) Let the joint *pdf* of X and Y be defined as $f_{XY}(x,y)=ce^{-\alpha x-\beta y}$, $x,\ y>0$. (a) determine the value of c, (b) find the marginal *pdf*s of X and Y, (c) find the $E[X]$, $Var[X]$ and $E[Y]$, $Var[Y]$ (d) find the correlation $E[XY]$, and (e) find the covariance $Cov(X,Y)$.

4. (12%) A binary transmission system transmits a signal X ($X=-1$ with probability p and $X=1$ with the one of $1-p$). The received signal is $Y=X+N$ where noise N has zero-mean Gaussian distribution with variance σ^2. (a) Find the conditional *pdf* of Y given the input value: $f_Y(y|X=1)$ and $f_Y(y|X=-1)$. (b) Find the *pdf* $f_Y(y)$. (c)Assume that $p=0.75$ and $\sigma^2=16$, what is the probability of $P(Y>0)$, $P(Y>0|X=-1)$, and $P(Y<0|X=1)$.

5. (12%) Find the eigenfunction expansion of the given function $f(x)=\begin{cases}-1, & 0\le x\le 2\\ 1, & 2<x\le 4\end{cases}$ by using the eigenfunctions of the Strum-Liouville problem $y''+\lambda y=0,\ y'(0)=y(4)=0$. Determine what the

eigenfunction expansion converges to on the interval.

6. (15%) (a) Prove the following form of Parseval theorem $\int_{-\infty}^{\infty}\left|f(t)\right|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|F(\omega)\right|^2 d\omega$ where $F(\omega)$ is the Fourier transform of $f(t)$. (b) Find the Fourier transform of $f(t)=\begin{cases} a, & |t|\le a \\ 0, & |t|>a \end{cases}$. (c) Use the Parseval theorem to evaluate the integral $\int_{-\infty}^{\infty}\left(\frac{sin(\omega a)}{\omega a}\right)^2 d\omega$.

7. (12%) Use the Laplace transform to solve the system

$$\begin{aligned} &y_1' - 2y_2' + 3y_1 = 0 \\ &y_1 - 4y_2' + 3y_3' = t \\ &y_1 - 2y_2' + 3y_3' = -1 \\ &y_1(0) = y_2(0) = y_3(0) = 0 \end{aligned}$$

8. (10%) Consider the differential equation $\frac{dy}{dx}=\frac{ax+by+c}{Ax+By+C}$ with $aB \ne bA$. (a) Solve the differential equation $\frac{dy}{dx}=\frac{x-y-1}{x+y+3}$ by finding h and k so that the substitutions $x=u+h$, $y=v+k$ transform it into the homogeneous equation $\frac{dv}{du}=\frac{u-v}{u+v}$. (b) Use this method to solve the differential equation $\frac{dy}{dx}=\frac{x-y-1}{x+y+3}$.