國立彰化師範大學 98 學年度<u>碩</u>士班招生考試試題

系所：電信工程學研究所　　科目：工程數學

☆☆請在答案紙上作答☆☆　　

1. (20%) Legendre's equation is $\left(1-t^2\right)y''-2ty'+\mu(\mu+1)y=0$. This equation has a power series solution of the form $y(t)=\sum_{n=0}^{\infty}a_n t^n$ that is guaranteed to be absolutely convergent in the interval $-1<t<1$.
   (a) Find the recurrence relation for the coefficients of the power series.
   (b) Argue, when $\mu=N$ is a nonnegative integer, that Legendre's equation has a polynomial solution, $P_N(t)$.
   (c) Use the recurrence relation and the requirement that $P_N(1)=1$ to determine the first three Legendre polynomials, $P_1(t)$, $P_2(t)$, $P_3(t)$.

2. (20%) Consider the boundary value problem $y''+\lambda y=0$, $y'(0)=0$, and $y'(1)+y(1)=0$.
   (a) Find the eigenvalues λ_n and **normalized** eigenfunctions $\phi_n(x)$.
   (b) Expand the function $f(x)=1-x,\ 0\le x\le 1$ in terms of the normalized eigenfunctions $\phi_n(x)$ found above.

3. (15%) Consider the initial value problem $y''+by'+cy=f(x)$, $0<t<\infty$, $y(0)=y_a$ and $y'(0)=y_b$. The input function $f(t)=e^{-t}$ and the Laplace transform of the output function $y(t)$ is $Y(s)=\dfrac{s^2+s+1}{(s+1)\left(s^2+4\right)}$. Determine the constants of b, c, y_a, y_b.

4. (15%) Prove that for $a>0$ and $x^{\beta}\ge 0$, $\int_0^{\infty}\dfrac{x^{\beta}}{(x+a)^2}dx=\dfrac{\pi}{a}\dfrac{\beta a^{\beta}}{\sin(\beta\pi)}$.

5. (15%) Let matrix $\mathbf{A}=\begin{bmatrix}-1 & 2 & 1 & -1 & -2\\ 2 & -4 & 1 & 5 & 7\\ 2 & -4 & -3 & 1 & 3\end{bmatrix}$. Determine the dimension of the **column space**, **null space** and **row space** of $\mathbf{A}$. Let $\mathbf{A}^T$ be the transpose of $\mathbf{A}$. What is the dimension of **null space** of $\mathbf{A}^T$ **?**

6. (15%) Let $\mathbf{A}=\begin{bmatrix}1 & 2 & 1 & -1\\ 2 & 4 & 1 & 1\\ 3 & 2 & -1 & -2\\ 2 & 5 & 3 & 0\end{bmatrix}$.

   (a) Find a permutation matrix $\mathbf{P}$ such that $\mathbf{PA}$ has a *LU* decomposition.

   (b) What's the *LU* decomposition of $\mathbf{PA}$ **?**