系所：顯示技術研究所　　　　　　　　　　　　　　　　科目:工程數學

☆☆請在答案紙上作答☆☆　　　　　　　　　　　　　　共 **1** 頁，第 **1** 頁

1. (25%)
   (a) Solve the following differential equation (15%)

   $$y'' + 4y' + 4y = 3xe^{-2x} + \sin(2x+2)$$

   (b) Solve the following differential equation(10%)

   $$x^{-3}y' - x^{-2}y - y^2 = 0$$

2. (25%)
   Solve the initial value problem for the system

   $$\begin{cases} y_1'' + 8y_1 - 2y_2 = 1 - u(t-2) \\ y_2'' - 2y_1 + 5y_2 = 0 \\ \quad y_1(0) = y_1'(0) = y_2(0) = y_2'(0) = 0 \end{cases}$$

   　　　　Where $u(t\text{-}2)$ is an unit step function or Heaviside function

3. (25%)

   Evaluate $\iint\limits_{S} F \cdot n dA$

   by(a) direct method (surface integral)(15%)
   　(b) the divergence theorem (10%)

   where

   $F = 4xzi - y^2 j + yzk,$
   S is the surface of a cube (正方形) bounded by x=0, x=1, y=0, y=1, z=0, z=1,
   n is the unit outer normal vector.

4. (25%)

   (a) Find the Fourier series of the function $f(x)$ where $f(x) = x^2, -\pi < x < \pi$ (10%)

   (b) use the results in (a) to prove $\dfrac{\pi^2}{6} = \sum\limits_{n=1}^{\infty} \dfrac{1}{n^2} = 1 + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \dfrac{1}{4^2} + \ldots\ldots\ldots\ldots$ (10%)

   (c) use the results in (a) to calculate $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{n^2} = ?$(5%)