國立彰化師範大學 99 學年度碩士班招生考試試題

系所：科學教育研究所　　組別：甲組　　科目：普通數學

☆☆請在答案紙上作答☆☆　　

1. Find the limit of the function $f(x) = x\left[\frac{1}{x}\right]$ when x approaches 0. Justify your answer! (Here the symbol $[a]$ denotes the greatest integer less than or equal to a.) (13 %)

2. Show that $|\sin x - \sin y| \leq |x - y|$, for all real numbers x and y. (12 %)

3. Let $f(x) = \int_2^{x^2} \frac{t^{3/2}}{\sqrt{t^2+17}}\,dt$. Find $\frac{df(x)}{dx}$. (13 %)

4. If $f(x) \geq 0$ on the interval $[a, b]$ and $\int_a^b f(x)dx = 0$, is it true that $f(x) = 0$ for all x in $[a, b]$? Justify your answer! (Prove it or give a counterexample.) (12 %)

5. Let V be the vector space of all the functions $f : \mathbb{R} \to \mathbb{R}$, and let U be its subspace of all differentiable functions. Let $T : U \to V$ be defined by $T(f) = f'$, the derivative of f. Show that T is a linear transformation of U into V. (10 %)

6. Let V and V' be vector spaces, and let $T : V \to V'$ be a linear transformation. If W' is a subspace of V', show that the set $\{v \in V \mid T(v) \in W'\}$ is a subspace of V. (10 %)

7. Let T_1 and T_2 be two linear transformations from $\mathbb{R}^n$ into $\mathbb{R}^m$, and let B be a basis for $\mathbb{R}^n$. If $T_1(b) = T_2(b)$, for every $b \in B$, show that $T_1 = T_2$. (10 %)

8. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that reflects vectors in the line $x + 2y = 0$ (以直線 $x + 2y = 0$ 為對稱軸做鏡射). Find the eigenvalues, the corresponding eigenvectors, and the standard matrix representation of T. (20 %)