國立彰化師範大學 100 學年度碩士班招生考試試題

系所：科學教育研究所　　組別：甲組　　科目：普通數學(含微積分與線性代數)

☆☆請在答案紙上作答☆☆　　　　　　　　　　　　　　　　　　**共 1 頁，第 1 頁**

1. (a) Prove the Squeezing Theorem: Let f, g and h be real functions with $g(x) \leq f(x) \leq h(x)$ for all $x \in I$, where I is an open interval in $\mathbf{R}$ containing c. If $\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L$, then $\lim_{x\to c} f(x) = L$. (10%)

   (b) Compute the limit: $\lim_{x\to\infty} \frac{\sin x}{x}$. (5%)

   (c) Compute the limit: $\lim_{x\to 0} \frac{\sin x}{x}$. (5%)

2. Sketch the graph of $y = \frac{x^3-x^2-4}{x-1}$ by analysing its first and second derivatives. Also, indicate the asymptotic line of the graph. (15%)

3. Derive the area of the surface of a sphere of radius r. (15%)

4. Define

$$T : M_{2\times 2}(\mathbf{R}) \to P_2(\mathbf{R})$$

by

$$T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (a+c) + bx + dx^2,$$

where $M_{2\times 2}(\mathbf{R})$ denotes the vector space of all 2×2 real matrices and $P_2(\mathbf{R})$ the vector space of all real polynomials with order at most 2. Let

$$\beta = \left\{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right\} \text{ and } \gamma = \left\{1, x, x^2\right\},$$

be the standard bases of $M_{2\times 2}(\mathbf{R})$ and $P_2(\mathbf{R})$, respectively.

   (a) Show that T is linear. (10%)

   (b) Compute the matrix representation of T with respect to the ordered bases β and γ, i.e. $[T]_\beta^\gamma$. (10%)

5. Let

$$A = \begin{pmatrix} 1 & -4 & -5 \\ -1 & 2 & -1 \\ 3 & 4 & 9 \end{pmatrix}.$$

   (a) Show that A is invertible. (5%)

   (b) Compute A^{-1}. (10%)

   (c) Compute A^n for $n \in \mathbf{N}$ and justify if $\lim_{n\to\infty} A^n$ exists in $M_{3\times 3}(\mathbf{R})$ accordingly. (15%)