

# 國立彰化師範大學 98 學年度碩士班招生考試試題

系所：科學教育研究所

組別：甲組

科目：普通數學(含微積分及線性代數)

☆☆請在答案紙上作答☆☆

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(1) Suppose  $W_1 = \left\{ \begin{bmatrix} x & 2x+2y \\ 0 & y \end{bmatrix} : x, y \in \mathfrak{R} \right\}$  and  $W_2 = \left\{ \begin{bmatrix} x & y \\ -y & -x \end{bmatrix} : x, y \in \mathfrak{R} \right\}$ . Find the

dimensions of  $W_1$ ,  $W_2$ ,  $W_1 + W_2$ , and  $W_1 \cap W_2$ . (20%)

(2) If matrix  $A$  is similar to matrix  $B$ , show that:

(a) What is the definition of “similar”? (10%)

(b) The trace of  $A^T$  is equal to that of  $B^T$ . (10%)

(3) Let  $A = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$ . Find  $A^{10}$  and  $\lim_{n \rightarrow \infty} (A^{-1})^n$ . (20%)

(4) (a) State without proof a version of the Fundamental Theorem of Calculus. (10%)

(b) Find  $\frac{d}{dx} \int_{\frac{\pi}{2}}^{x^3} \cos t \, dt$ . (10%)

(5) Determine whether the following series converge or diverge. Please give detailed reasons for your answers. (20%)

(a)  $\sum_{n=1}^{\infty} \frac{n!}{n^n}$  (b)  $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{8n^2 - 5n}}$