國立彰化師範大學 97 學年度碩士班招生考試試題

系所：科學教育研究所碩士班　　組別：甲組　　科目：普通數學（含微積分及線性代數）

☆☆請在答案紙上作答☆☆　　共 1 頁，第 1 頁

Part I. Linear Algebra (60%)

1. Find the Jordan form *J* of the matrix *A* below together with a non-singular matrix *P* such that $P^{-1}AP = J$.

$$A=\begin{bmatrix} 3 & 0 & 1 \\ -4 & 1 & -2 \\ -4 & 0 & -1 \end{bmatrix}$$ (20%)

2. Find the matrix of the linear transformation $T(x_1, x_2, x_3) = (4x_1 + x_2 - x_3, x_1 + 3x_3, x_2 + 2x_3)^T$ with respect to the basis $(1, 1, 1)^T$, $(1, 0, 1)^T$, $(0, 1, 1)^T$. (20%)

3. Suppose $A \in F^{n\times n}$ has the property that $A = A^{-1}$. Show that if λ is an eigenvalue of *A*, then so is λ^{-1}. (20%)

Part II. Calculus (40%)

1. Let *f* be a function of *x* and *y* which has continuous first and second partial derivatives throughout some set *D* in the plane. Suppose that $f_{xy}(x, y) = 0$ for all $(x, y) \in D$. What can you conclude about *f*? (20%)

2. Find *H'*(2) given that

$$H(x) = \int_{2x}^{x^3-4} \frac{x}{1+\sqrt{t}}\, dt.$$ (20%)