國立彰化師範大學 102 學年度碩士班招生考試試題

系所：數學系　　組別：甲組　　科目：機率與統計

☆☆請在答案紙上作答☆☆　　

1. Suppose X and Y have the joint p.d.f. (30%)

$$f(x,y)=\begin{cases} ce^{x-2y} & \text{if } 0\le x<y<\infty; \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find c.
(b) Compute $P(X+Y\le 1)$.
(c) Compute $E(X\mid Y=1)$.

2. Let X and Y be independent random variables taking values in $S=\{1,2,3\}$. Let $Z=X+Y$ and suppose $P(Z=2)=P(Z=6)=\frac{1}{9}$, $P(Z=3)=P(Z=5)=\frac{2}{9}$ and $P(Z=4)=\frac{3}{9}$. Prove that $P(X=k)=P(Y=k)=\frac{1}{3}$ for $k=1,2,3$. (20%)

3. Consider the following regression model

$$Y_i=a+b_1X_{i,1}+b_2X_{i,2}+\cdots+b_kX_{i,k}+\sigma\varepsilon_i,$$

where σ is a positive constant, $Y_i, i=1,2,\cdots,n$, are responses, $X_{i,j}$, $i=1,\cdots,n$, $j=1,\cdots,k$ are covariates and ε_i, $i=1,2,\cdots,n$, are i.i.d. random variables with standard normal distribution. Do the maximum likelihood estimators and least square estimators of $a,b_1,b_2,\cdots,b_k$ have the same form? Why? (15%)

4. Does music really help increasing the efficiency of doing homework for elementary schoolboys? Below is a data recording the time spent in homework for ten schoolboys exposed to classical music and other ten schoolboys not exposed to classical music. Please conduct a hypothesis testing using $\alpha=0.05$, to test if listening to classical music did help reducing the time (in minutes) doing homework. (Provided that the two groups have the same variance) (20%)

	Time spent									
Boys with music	20	19	15	25	30	14	16	17	15	17
Boys without music	22	21	18	28	33	13	15	28	32	15

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02
df								
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528

5. We would like to compare the effectiveness of two teaching methods. A number of schoolboys are to be divided into two groups: group 1 and group 2. Suppose that we recruit the schoolboys in some area with the same variance $\sigma = 2$ of scores (scaled from 1 to 10), and randomly allocate the same number n of schoolboys into each group. For a confidence level of 95%, how large the number n of schoolboys in each group is necessary? (15%)